Expanding plasmas from Anti de Sitter black holes

(based on 1609.07116 [hep-th])

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Objective

- Long-term goal:

Understand the far-from-equilibrium behavior of QFTs.

- HERE:

A good starting point: **a QFT plasma undergoing (spatial) expansion**.

- Simplifying assumption #1:

Put the QFT in an **expanding FLRW background**:

$$ds^2 = -dt^2 + a(t)^2 d\Omega_k^2$$

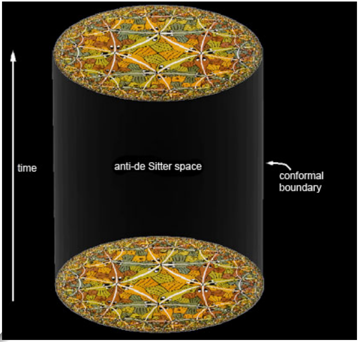
Even a locally static fluid in FLRW has a nonzero expansion rate since spacetime itself expands,

$$u^\mu = (1, 0, 0, 0) \quad \longrightarrow \quad \nabla_\mu u^\mu = 3\frac{\dot{a}(t)}{a(t)}$$

Statement of the problem

▶ Simplifying assumption #2:

Approach the problem using **holography**, i.e.,
consider **strongly coupled CFT with known AdS gravity dual**



▶ **The Problem:**

Find slicing of static bulk metrics such that

AdS bdry = FLRW

Begin with generic AdS_5 BH written in standard (t, r) coordinates:

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + \Sigma(r)^2 d\Omega_k^2$$

- f and Σ arbitrary except for

$$f(r) \sim \frac{r^2}{L^2}, \ \Sigma(r) \sim \frac{r}{L} \quad \text{for } r \to \infty \qquad \text{(AdS asymptotics)}$$
$$f(r_h) = 0 \qquad \text{(event horizon)}$$

- $k = +1, 0, -1$ all allowed
- Eventual supporting matter fields are OK (scalar hair, charge, etc)

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + \Sigma(r)^2 d\Omega_k^2$$

- **STEP 1:** Go to EF coords (v, r), i.e.,

$$dv = dt + f(r)^{-1}dr$$

- **STEP 2:** Introduce “fake” time dependence by redefining v as

$$dv = \frac{dV}{a(V)}$$

- **STEP 3:** Redefine radial coordinate as $r = Ra(V)$ so as to put the metric back in EF-like coordinates (V, R):

$$\boxed{ds^2 = 2dV dR - \left[\frac{f(Ra)}{a^2} - 2R\frac{\dot{a}}{a}\right] dV^2 + \Sigma(Ra)^2 d\Omega_k^2}$$

$$\boxed{ds^2 = 2dV\,dR - \left[\frac{f(Ra)}{a^2} - 2R\frac{\dot{a}}{a}\right] dV^2 + \Sigma(Ra)^2 d\Omega_k^2}$$

- At $R = \infty$:

$$ds^2 = \frac{R^2}{L^2}\left[-dV^2 + a(V)^2 d\Omega_k^2\right],$$

  so we have achieved our goal of setting the ∂AdS to be FLRW

- Holographic dual of a CFT in a FLRW background
- Using it we can now extract info about how the CFT plasma expands,

$$\langle T^{\mu\nu}(V)\rangle, \ldots$$

Example 1: AdS-Schwarzschild

Einstein-($\Lambda < 0$) action: $S = \frac{1}{16\pi G_5} \int d^5x\sqrt{-g}\left[R + \frac{12}{L^2}\right]$
Static solution: $f(r) = \frac{r^2}{L^2}\left(1 + \frac{kL^2}{r^2} - \frac{M}{r^4}\right) \qquad \Sigma(r) = \frac{r}{L}$

$$\boxed{ds^2 = 2dV\,dR - \left[\frac{R^2}{L^2}\left(1 + \frac{kL^2}{R^2a^2} - \frac{M}{R^4a^4}\right) - 2R\frac{\dot{a}}{a}\right]dV^2 + \frac{R^2a^2}{L^2}d\Omega_k^2}$$

Holographic dual of $\mathcal{N} = 4$ SYM plasma in FLRW spacetime.

The plasma has a local temperature

$$T(V) = \frac{kL^2 + 2r_h^2}{2\pi L^2 r_h a} \ .$$

$$
\begin{aligned}
\langle T_{\tau\tau}\rangle &\equiv \mathcal{E} = \frac{3(\dot{a}^2+k)^2+12M}{64\pi G_5 a^4} \\
\langle T^{i}{}_{i}\rangle &\equiv \mathcal{P} = \frac{(\dot{a}^2+k)^2+4M-4a\ddot{a}(\dot{a}^2+k)}{64\pi G_5 a^4}
\end{aligned}
$$

Use $G_5 = \frac{\pi L^3}{2N_c^2}$ and $M = M(T)$ to put in pure CFT language. For ex., for $k = 0$

$$
\begin{aligned}
\mathcal{E} &= \frac{3N_c^2 T^4}{8} + \frac{3N_c^2}{32\pi^2}\frac{\dot{a}^4}{a^4} \\
\mathcal{P} &= \frac{\mathcal{E}}{3} - \frac{N_c^2}{8\pi^2}\frac{\ddot{a}\dot{a}^2}{a^3} \ .
\end{aligned}
$$

Nontrivial $a(t)$ breaks conformal symmetry and leads to a **conformal anomaly**

$$
\langle T^{\mu}{}_{\mu}\rangle = 3\mathcal{P} - \mathcal{E} = -\frac{3\ddot{a}(\dot{a}^2+k)}{16\pi G_5 a^3}
$$

Example 2: AdS-Gauss-Bonnet

EGB-($\Lambda < 0$) action: $S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[R + \frac{12}{L^2} + \frac{L^2}{2} \lambda_{\text{GB}} (R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2) \right]$

- Holographic dual of a CFT with two central charges c, b

$$c = \frac{\pi L^3_{\text{AdS}}}{8G_5} \sqrt{1 - 4\lambda_{\text{GB}}} \qquad b = \frac{\pi L^3_{\text{AdS}}}{8G_5} \left(-2 + 3\sqrt{1 - 4\lambda_{\text{GB}}} \right)$$

- $\mathcal{E}$ and $\mathcal{P}$ of the expanding plasma:

$$\begin{aligned} \mathcal{E} &= \frac{3(\dot{a}^2 + k)^2 + 12M}{64\pi G_5 a^4} - \frac{3\left[15(k + \dot{a}^2)^2 + 4M - 64a\ddot{a}(k + \dot{a}^2 - a\ddot{a})\right]}{128\pi G_5 a^4} \lambda_{\text{GB}} \\ \mathcal{P} &= \frac{(\dot{a}^2 + k)^2 + 4M - 4a\ddot{a}(k + \dot{a}^2)}{64\pi G_5 a^4} - \frac{15(k + \dot{a}^2)^2 + 4M - 4a\ddot{a}[31(k + \dot{a}^2) - 16a\ddot{a}]}{128\pi G_5 a^4} \lambda_{\text{GB}} \end{aligned}$$

- Conformal anomaly $\langle T^\mu_{\ \mu} \rangle = 3\mathcal{P} - \mathcal{E} = -\left(1 - \frac{15}{2}\lambda_{\text{GB}} \right) \frac{3\ddot{a}(k+\dot{a}^2)}{16\pi G_5 a^3}$

Example 3: AdS-Reissner-Nordström

Einstein-Maxwell-($\Lambda < 0$) action:

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[R + \frac{12}{L^2} - \frac{1}{4} F_{ab} F^{ab} \right]$$

Static solution (planar horizon $k = 0$):

$$\begin{aligned} f(r) &= \frac{r^2}{L^2} \left(1 - \frac{M}{r^4} + \frac{Q^2}{r^6} \right) \qquad \Sigma(r) = \frac{r}{L} \\ A_a dx^a &= \mu \left(1 - \frac{r_h^2}{r^2} \right) dt \end{aligned}$$

- Dual to a CFT plasma with temperature T and chemical potential μ
- Interesting also to illustrate how our foliation works in the presence of matter fields coupled to the metric

FLRW foliation dual to the expanding plasma

$$ds^2 = 2dV\,dR - \left[\frac{R^2}{L^2}\left(1 - \frac{M}{R^4a^4} + \frac{Q^2}{R^6a^6}\right) - 2R\frac{\dot{a}}{a}\right]dV^2 + \frac{R^2a^2}{L^2}d\mathbf{x}^2$$

$$A_a dx^a = \mu\left(1 - \frac{r_h^2}{R^2a^2}\right)\left[\left(\frac{1}{a} - \frac{L^2\dot{a}/Ra^2}{1 - \frac{M}{R^4a^4} + \frac{Q^2}{R^6a^6}}\right)dV - \frac{L^2/R^2a}{1 - \frac{M}{R^4a^4} + \frac{Q^2}{R^6a^6}}dR\right]$$

The plasma is subject to a time-dependent chemical potential $\tilde{\mu}(V) \equiv \frac{\mu}{a(V)}$:

$$A_\mu dx^\mu\big|_{\text{bdry } R=\infty} = \frac{\mu}{a}dV \ .$$

Cool lesson:

Flat space plasma subject to a "quench" $\tilde{\mu}(V)$ in the chemical potential experiences nonequilibrium dynamics similar to a cosmological evolution with

$$a(V) \sim \tilde{\mu}(V)^{-1}$$

Energy density $\mathcal{E} \equiv \langle T_{\tau\tau} \rangle$, pressure $\mathcal{P} \equiv \langle T^i{}_i \rangle$, and charge density $\mathcal{Q} \equiv \left\langle J^\tau \right\rangle$

$$
\begin{aligned}
\mathcal{E} &= \frac{3\dot{a}^4 + 12 r_h^2 (r_h^2 + \frac{1}{3}\mu^2)}{64\pi G_5 a^4} \\
\mathcal{P} &= \frac{\dot{a}^4 + 4 r_h^2 (r_h^2 + \frac{1}{3}\mu^2) - 4a\ddot{a}\dot{a}^2}{64\pi G_5 a^4} \\
\mathcal{Q} &= \frac{\mu\left(2r_h^2 + 2\dot{a}^2 + a\ddot{a}\right)}{16\pi G_5 a^3} \, .
\end{aligned}
$$

Conformal anomaly is the same as in SAdS: $\langle T^\mu{}_\mu \rangle = -\frac{3\ddot{a}(\dot{a}^2+k)}{16\pi G_5 a^3}$

$T^{\mu\nu}(V)$ is conserved while charge density $\mathcal{Q}$ is not:

$$
\begin{aligned}
\nabla_\mu \langle T^{\mu\nu} \rangle &= 0 \\
\nabla_\mu \langle J^\mu \rangle &= \frac{\mu}{2a^3}\left(-3\dot{a}\ddot{a} + a\dddot{a}\right)
\end{aligned}
$$

Entropy production

$$\boxed{ds^2 = 2dVdR - \left[\frac{f(Ra)}{a^2} - 2R\frac{\dot{a}}{a}\right] dV^2 + \Sigma(Ra)^2 d\Omega_k^2}$$

- **Apparent horizon** $R_h(V)$ defined by $f(R_h a)\Sigma'(R_h a) = 0$.
- **Nonequilibrium entropy density** of the dual expanding plasma can be associated with the BH entropy of the apparent horizon

$$s_{\text{BH}} = \frac{\Sigma(R_h a)^3}{4G_5}$$

  - For **conformal** plasmas: $\frac{ds}{dV} = 0$, i.e., **no entropy production** by the plasma (no dissipation due to $\zeta = 0$)
  - For **nonconformal** plasmas: $\frac{ds}{dV} > 0$ due to dissipative effects.

Conclusions and Outlook

- New slicing of generic AdS BHs such that ∂AdS $=$ FLRW.
  - EF coordinates make the task trivial
  - Applicable to a variety of solutions, supported by external fields or not
  - New arena to explore nonequilibrium dynamics of CFTs **analytically**
- Quenches (time-dependent couplings) arise naturally when analyzing BHs supported by matter fields:

Gauge field	$A_\mu dx^\mu(r) \sim \mu dv + \cdots$	$\longrightarrow$	$A_\mu dx^\mu(Ra) \sim \frac{\mu}{a} dV + \cdots$
Scalar hair	$\phi(r) \sim J r^{\Delta - d} + \cdots$	$\longrightarrow$	$\phi(Ra) \sim J a^{\Delta - d} R^{\Delta - d} + \cdots$

- Cosmology?

  $\vdots$

Thank you! ☺

Quantum quench $\tilde{J}(t) \equiv Ja(t)^{\Delta-d}$ of a relevant $\mathcal{O}_\Delta$:

$$S_{\text{QFT}} = S_{\text{CFT}} + \int_{\text{flat}} d^d x Ja(t)^{\Delta-d} \mathcal{O}_\Delta^{(\text{flat})}(t,x)$$

Weyl rescaling $ds^2\Big|_{\text{flat}} = a(t)^{-2} ds^2\Big|_{\text{FLRW}}$:

- $\sqrt{-g} = a(t)^{-d}$
- $\mathcal{O}_\Delta^{(\text{FLRW})}(\tau,x) = a(t)^\Delta \mathcal{O}_\Delta^{(\text{flat})}(t,x)$

$$S_{\text{QFT}} = S_{\text{CFT}} + \int_{\text{FLRW}} d^d x \sqrt{-g} J \mathcal{O}_\Delta^{(\text{FLRW})}(\tau,x)$$

$\Rightarrow$ **Constant deformation J but in a FLRW spacetime!**