Some things I wish to emphasize

We discussed dualities in general and in some examples. A "common" "denominator" appears.

In principle the two dual descriptions

— have different degrees of freedom

[the fields used in both descriptions are not necessarily the same — but they could be the same]

— But the global symmetries are the same

[local symmetries? no!]

In the case of bosonization I should have emphasized one

Bosonic | Fermionic

$$\epsilon_{\mu\nu}\partial_\nu \Phi \longleftrightarrow \bar{\Psi}\gamma_\mu\Psi$$

Topological current | Noether current

— "Inverts" perturbative → non perturbative → $\boxed{\text{Classical} \leftrightarrow \text{Quantum}}$

In the Maxwell → Maxwell case no interesting global symmetries

Let me tell you about a Duality that exemplifies these points.

This duality was conjectured by Seiberg in 1994. It is NOT proven.

It cannot be obtained with the procedure of gauging a global symmetry + imposing zero $F_{\mu\nu}$ + integrating "the other way around"

Mostly because it involves a non-Abelian gauge field

$\rightsquigarrow$ integrals of the form $\int dx\, e^{\frac{i}{\hbar}(kx^2+px^4)} = ?$

$\mathcal{N}=1$ SQCD

QCD $\mathcal{L} = \mathrm{Tr}\{F_{\mu\nu}^2 + \bar{\Psi}\not{D}\Psi\} + \tilde{\Psi}\not{D}\tilde{\Psi}\}$.

$\mathcal{N}=1$

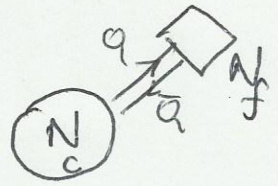
$$\left.\begin{aligned} A_\mu &\to A_\mu,\ \lambda \\ \Psi_f &\to \phi_f,\ \Psi_f \\ \tilde{\Psi}_f &\to \tilde{\phi}_f,\ \Psi_f \end{aligned}\right\} \quad \begin{aligned} \mathcal{L} = {} & F_{\mu\nu}^2 + \bar{\lambda}\not{D}\lambda \\ & + |D_\mu\phi|^2 + |D_\mu\tilde{\phi}|^2 \\ & + \bar{\Psi}\not{D}\Psi + \tilde{\Psi}\not{D}\tilde{\Psi} \\ & + \underbrace{V(\lambda,\Psi,\tilde{\Psi},\phi,\tilde{\phi})}_{\text{encodes the susy properties}} \end{aligned}$$

$$\mathcal{L} = \int d^4\theta\, Q^\dagger e^V Q + \tilde{Q}^\dagger e^{-V}\tilde{Q} + \int d^2\theta\, W_\alpha W^\alpha + \int d^2\theta\, W(Q,\tilde{Q})$$

Seiberg Duality

$\mathcal{N}=1$ SUSY GFT

gauge $SU(N_c)$; $W_\alpha = (A_\mu, \lambda)$



flavor $SU(N_f)$; $Q = (q, \Psi_q)$
$\bar{Q} = (\bar{q}, \Psi_{\bar{q}})$

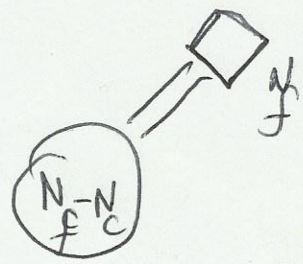
$$\mathcal{L} \sim \mathrm{Tr}\left\{ -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2}|D_\mu q|^2 - \frac{1}{2}|D_\mu \bar{q}|^2 - i\,\bar{\Psi}_q \not{D} \Psi_q - i\,\bar{\Psi}_{\bar{q}} \not{D} \Psi_{\bar{q}} - i\,\bar{\lambda} \not{D} \lambda \right\}$$

	$SU(N_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)_R$	$U(1)_B$
W_α	adj	1	1	1	0
Q	$\square$	N	1		1
$\bar{Q}$	$\bar{\square}$	1	N		-1

$\Updownarrow$ In the IR $\left(N_f < \frac{3}{2} N_c,\ N_c + 1 < N_f \right)$

gauge $SU(N_f - N_c)$
flavor $SU(N_f)$

$w_\alpha = (\hat{\lambda}, \hat{a}_\mu)$
$q = (q, \chi_q)$
$\bar{q} = (\bar{q}, \bar{\chi}_q)$
$M = (m, \chi_m)$



$$\mathcal{L} \sim \mathcal{L}_{SQCD} + \bar{q} M q$$

In this example we see all the "features"

	Before	After
gauge symmetry	$SU(N_c)$	$SU(N_f - N_c)$
global "	$SU(N_f) \times SU(N_f) \times U(1)_B \times U(1)_R$	$\longrightarrow$ same
field content	$A_\mu, \lambda, \phi_q, \psi_q$	a_μ, χ
	$\tilde{\phi}_q, \tilde{\psi}_q$	q, f_q
		$\tilde{q}, \tilde{f}_q$
		$M = (m, f_m)$

But it was <u>conjectured</u> that both QFT describe the same IR Physics.

One duality that is <u>VERY special</u>

Maldacena Conjecture

- equivalence between a 4d QFT (no gravity) and a quantum theory of gravity in a given space time.

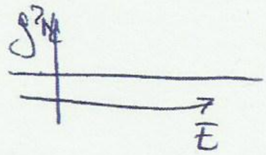
- $\overset{\text{QFT}}{\mathcal{N}=4}$ SYM with gauge group $SU(N_c)$ in 4-dim

  - $A^a_\mu,\ 4\times\lambda^a,\ 6\times\phi^a \qquad a: 1, \ldots N_c^2-1$

  $SO(6) \sim SU(4)$ global symmetry., 32 SUSY

  $SO(1,3) \longrightarrow SO(2,4)$ conformal

  coupling $\lambda = g^2 N_c$ = constant $\begin{cases} <1 \\ \geq 1 \end{cases}$



- Quantum gravity: IIB String theory on $AdS_5 \times S^5$.

$$ds^2 = \alpha' \left\{ \frac{u^2}{R^2}\left(dx^2_{1,3}\right) + \frac{R^2}{u^2} du^2 + R^2 d\Omega_5 \right\} \qquad R^4 = 4\pi^2 g_s N_c$$

$[u]$ = Energy

$$F_5 = N_c \left(\text{Vol } S^5 + \text{Vol } AdS_5\right) \ ; \quad \int F_5 = N_c$$

$SO(6) \longrightarrow S^5$, 32 SUSY ; parameters $\lambda = g_s N_c = g^2_{YM} N_c$

$SO(2,5) \longrightarrow AdS_5$

Swieca

~~Queseeo~~ Lecture 2

We have briefly presented the Maldacena Conjecture

or AdS/CFT duality.

$\mathcal{N}=4$ SYM $\qquad = \qquad$ IIB String theory on $AdS_5 \times S^5$.

a couple of immediate comments

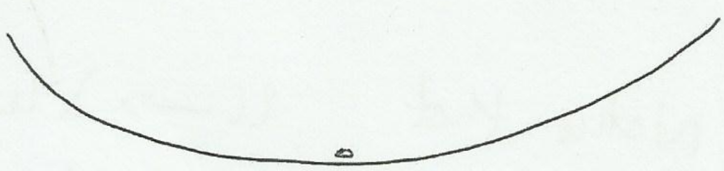
- Same global symmetries $\quad SO(6)$, $SO(2,4)$, 32 SUSY

- Same parameters
  always in limit $g_s \to 0$

$$\left[\begin{array}{lcl} g_s N_c & \leftrightarrow & g_{YM}^2 N_c \\ \frac{R^4}{\alpha'^2} = 4\pi g_s N_c & \longleftrightarrow & \lambda = g_{YM}^2 N_c \end{array}\right]$$

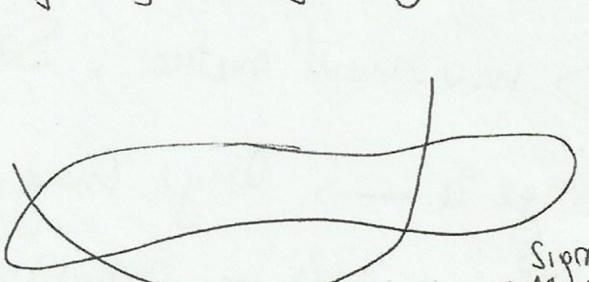
- Notice that $\left.\begin{array}{l}\alpha' \sim l_s^2 \\ R \sim \text{Lenght of Space}\end{array}\right] \longrightarrow \frac{R^4}{\alpha'^2} \begin{cases} \to \infty \\ \to 0 \end{cases}$

If $\frac{R^4}{\alpha'^2} \to \infty \quad \longrightarrow$



semiclassical ($g_s \to 0$)
and point particle limit ($\alpha' \to 0$)
of the full string theory $\equiv$ Supergravity

If $\frac{R^4}{\alpha'^2} \to 0 \quad \longrightarrow$



Semiclassical ($g_s \to 0$) but extended string $\to$ Sigma Model

Now, let us observe that in the

$g_s \to 0$ semiclassical

$R^2/\alpha' \to 0$ point particle limit of the string theory $\equiv$ $\lambda = g^2_{YM} N \to \infty$ GFT

→ quite complicated (strongly interacting) problem in 4d GFT

turns into a "classical" problem in String theory.

— Also viceversa

— Notice that if we write AdS_5 as

$$ds^2_{AdS_5} = \alpha' \left\{ \frac{u^2}{R^2} dx^2_{1,3} + \frac{R^2}{u^2} du^2 \right\} \to \boxed{\begin{array}{l} R^4 = 4\pi g_s N_c \\ u = r/\alpha' \end{array}}\ \boxed{[u] = \text{Energy}}$$

Notice that $\left.\begin{array}{l} u \to \lambda u \\ x \to x/\lambda \end{array}\right\}$ with $d\lambda = 0$

⇝ invariant metric. So this tells us that large u → small distance in x $(\lambda > 1)$

large u → large Energy in GFT. This will obviously play a role in non-conformal cases.

Finally notice that

$$\frac{R^4}{\alpha'^2} \sim \lambda \xrightarrow{\text{fixed } R} \alpha' \sim \frac{1}{\sqrt{\lambda}}$$

$$\frac{2\kappa_{10}^2}{\alpha'^4} = \frac{\lambda^2}{N^2} = g_s^2 \longrightarrow \boxed{G_{10\,N} \sim \frac{1}{N_c^2}}$$

$$\left.\begin{array}{l} \text{Quantum gravity corrections} \sim \dfrac{1}{N_c^2} \\[2ex] \text{Classical string correction} \sim \dfrac{1}{\sqrt{\lambda}} \end{array}\right\} \begin{array}{l} \text{large } N_c \to \text{kills quantum gravity} \\[2ex] \text{large } \lambda \text{ kills classical string corrections} \end{array}$$

Operative version of the Maldacena Conjecture

for a QFT person

$$Z_{\mathcal{N}=4\,SYM}[J_i] = \int DA_\mu^a D\vec{\phi}^a D\vec{\lambda}^a D\text{ghost}\ \exp\left[-\int \mathcal{L}_{\mathcal{N}=4\,SYM}\, d^4x + \int J_i\, O^i\right]$$

is the quantity of Interest.

The proposal by Gubser-Klebanov-Polyakov (1998)
Witten

is that

$$Z_{\mathcal{N}=4\,SYM}[J_i] \overset{\text{conjecture}}{\equiv} \underset{\substack{\text{IIB String theory}\\ \text{on } AdS_5\times S^5\\ \text{with boundary conds } J_i}}{Z} \left[\Phi_i \xrightarrow[\text{boundary}]{} J_i\right]$$

for an string theorist studying string on $AdS_5 \times S^5$

String fields

$$Z = \int D\Phi_i \, e^{-S_{IIB}} \Big|_{\text{bc that } \Phi_i \to J_i \text{ on boundary}}$$

bc that

$\Phi_i \to J_i$ on boundary.

Boundary of $AdS_5 \times S^5$

$$ds^2 = \alpha' u^2 \left\{ \frac{dx_{1,3}^2}{R^2} + \frac{R^2 du^2}{u^4} + \frac{R^2}{u^2} d\Omega_5 \right\}$$

$$\lim_{u \to \infty} ds^2 = dx_{1,3}^2$$

Both $Z_{QFT\,SYM}[J_i]$ and $Z_{IIB\,String}[\Phi_i \to J_i]$ are very

difficult to compute. But if we are for example in the semiclassical $g_s \to 0$ and point particle limit $\alpha' \to 0$ of the string one may approximate conjecture

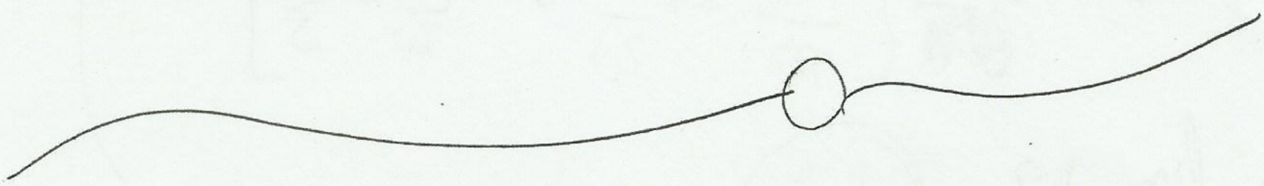
$$\underbrace{Z_{N=4\,SYM}[J_i]}_{\substack{\text{Very "interesting"} \\ \text{quantity}}} = \underbrace{e^{-S_{IIB}[\Phi \to J_i]}}_{\text{classical calculation}}$$

complicated

$\lambda = R^4/\alpha'^2 \to \infty$

$g_s \to 0 \Rightarrow N_c \to \infty$

"Simple"



Let us start one example (Maldacena 1998, Rey–Yee 1998)

Wilson loop

Conjecture

$$\langle W\rangle = \langle P\, e^{\oint_C A_\mu dx^\mu} \rangle \equiv Z_{\text{string}}[\;\;]$$

with BC

$$\sim e^{-S_{\text{string}}^{\text{with BC}}}$$

$$S_{\text{string}} = S_{NG} = \frac{1}{2\pi\alpha'}\int d^2\sigma \sqrt{\det g_{\alpha\beta}}$$

$$\boxed{g_{\alpha\beta} = G_{\mu\nu}\,\partial_\alpha X^\mu\, \partial_\beta X^\nu .}$$

Calcule en $\begin{cases} \text{any metric} \\ AdS_5 \end{cases}$ (Sonnenschein 1999)

Let us discuss the formalism of Wilson loops in a generic metric

$$ds^2 = -g_{tt}\,dt^2 + g_{xx}\,dx^2 + g_{\rho\rho}\,d\rho^2 + \dots$$

$$\left(g_{\alpha\beta} = G_{\mu\nu}\,\partial_\alpha X^\mu \partial_\beta X^\nu \quad \alpha,\beta = \sigma,\tau\right)$$

we propose a configuration (as before)

$$\left.\begin{array}{l} x = \sigma \\ \rho = \rho(\sigma) \\ t = \tau \end{array}\right\} \quad ds^2_{ind} = -g_{tt}\,d\tau^2 + \left(g_{xx} + g_{\rho\rho}\,\rho'^2\right) d\sigma^2$$

$$g_{\alpha\beta} = \begin{array}{c c} & \begin{array}{cc} \tau & \sigma \end{array} \\ \begin{array}{c} \tau \\ \sigma \end{array} & \begin{bmatrix} -g_{tt} & 0 \\ 0 & (g_{xx} + g_{\rho\rho}\,\rho'^2) \end{bmatrix} \end{array}$$

$$\longrightarrow -\det g_{\alpha\beta} = g_{tt}\left(g_{xx} + g_{\rho\rho}\,\rho'^2\right)$$

$$S_{NG} = \int \sqrt{g_{tt}\,g_{xx} + g_{tt}\,g_{\rho\rho}\,\rho'^2}\;\frac{d\sigma\,d\tau}{2\pi\alpha'} \rightsquigarrow \text{define} \quad \begin{array}{l} f^2 = g_{tt}\,g_{xx} \\ g^2 = g_{tt}\,g_{\rho\rho} \end{array}$$

$$S_{NG} = \frac{T}{2\pi\alpha'}\int d\sigma\, \sqrt{f^2_{(\rho)} + g^2_{(\rho)}\,\rho'^2} \longrightarrow$$ 1-d system in classical mechanics $\longrightarrow$ conserved Energy

with the Nambu-Goto Action

$$S_{NG} = \frac{T}{2\pi\alpha'} \int d\sigma \sqrt{f^2 + g^2 \rho'^2} \longrightarrow H = p_\rho \rho' - \mathcal{L}$$

$$H = -\frac{f^2}{\sqrt{f^2 + g^2\rho'^2}} \overset{\text{constant}}{=} -f(\rho_0) = -f_0$$

$\rightsquigarrow$ from here

$$\left[\frac{d\rho}{dx} = \frac{1}{f_0} \frac{f}{g} \sqrt{f^2 - f_0^2} \right] \rightsquigarrow \text{then the separation between } Q\bar{Q} \text{ pair is}$$

$$L_{Q\bar{Q}} = 2\int d\sigma = 2 \cdot \int_{\rho_0}^{\infty} d\rho \, \frac{f_0 \, g}{f\sqrt{f^2 - f_0^2}}$$

$$\boxed{L_{Q\bar{Q}}^{(\rho_0)} = 2 f_0 \int_{\rho_0}^{\infty} d\rho \, \frac{g}{f\sqrt{f^2 - f_0^2}}} \rightsquigarrow \text{Let us now compute the Energy}$$

$$E_{Q\bar{Q}} = \underbrace{\int d\sigma \sqrt{f^2 + g^2 \rho'^2}}_{\text{original } \frac{\text{NG Action}}{T}} - 2 \underbrace{\int_0^\infty g(\rho)\, d\rho}_{\text{mass of 2 straight strings } |\ |}$$

using as derived above $\left[\dfrac{d\rho \, f_0 g}{f\sqrt{f^2 - f_0^2}} = d\sigma \right]$

$$E_{Q\bar{Q}} = 2 f(\rho_0) \cdot \int_{\rho_0}^\infty \frac{g/f}{\sqrt{f^2 - f_0^2}}\, d\rho - 2 \int_0^\infty g\, d\rho$$

or using the expression for $L_{Q\bar{Q}}(\rho_0)$

Notice:

if $f(\rho_0) \neq 0$

$$\boxed{E_{Q\bar{Q}}(\rho_0) = f(\rho_0)\, L_{Q\bar{Q}}(\rho_0) + 2 \int_{\rho_0}^\infty \frac{g}{f}\left[\sqrt{f^2 - f_0^2} - f\right] d\rho - 2\int_0^{\rho_0} g\, d\rho}$$

$E \approx f(\rho_0)\, L_{Q\bar{Q}}$

~ confinement!