

INTRODUCTION TO NUCLEAR EFFECTIVE FIELD THEORIES[©]

bira

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"Simplicity, like everything else,
must be explained."

S. Weinberg



Exercise #1

For each lecture, seminar, colloquium, and poster where the holy words "effective field theory" are invoked, are they being used in the original sense?

(e.g. Is "leading order" leading?)



References:

U. van Kolck,
Effective Field Theories of Loosely Bound Nuclei,
in *The Euroschool on Exotic Beams, Vol. IV*
C. Scheidenberger and M. Pfützer (eds.),
Springer, Berlin Heidelberg (2014)
Lect. Notes Phys. **879** (2014) 123

U. van Kolck,
**Les Houches Lectures on Effective Field Theories
for Nuclear and (some) Atomic Physics,**
in *Proceedings of the 2017 Les Houches Summer School
on EFTs in Particle Physics and Cosmology,*
S. Davidson *et al.* (eds.),
Oxford University Press (to appear)
arXiv:1902.03141 (nucl-th)

□ Effective Field Theories

- ▶ Introduction
- ▶ What is effective
- ▶ Summary

□ EFT for Short-Range Forces

□ Two-Body System

□ Three-Body System

□ More-Body Systems

□ Long-range Forces

Wanted
Dead ♦ or ♦ Alive

FORMULATION OF NUCLEAR PHYSICS CONSISTENT WITH STANDARD MODEL (SM) OF PARTICLE PHYSICS

Reward

understanding emergence of complexity
at the most fundamental level:

nucleus made out of quarks and gluons interacting
strongly (QCD), yet exhibiting many regularities



use of nuclei as laboratories for physics beyond the SM

Beware

coupling constants not small: not an easy problem!

Not an easy problem...

“There are few problems in nuclear theoretical physics which have attracted more attention than that of trying to determine the fundamental interaction between two nucleons.

It is also true that scarcely ever has the world of physics owed so little to so many ...

... It is hard to believe that many of the authors are talking about the same problem or, in fact, that they know what the problem is.”

M. L. Goldberger

*Midwestern Conference on Theoretical
Physics, Purdue University, 1960*

Nuclear Physics

The canons of tradition

- I Nuclei are essentially made out of non-relativistic nucleons in two isospin states (protons and neutrons)
- II The interaction potential is mostly two-body, but there is evidence for smaller three-body forces
- III Isospin is a good symmetry, except for the Coulomb interaction, breaking in two-nucleon scattering lengths, and smaller effects
- IV External probes (e.g. photons) interact mainly with each nucleon, but there is evidence for smaller two-nucleon currents

but...

WHY?

Quantum Chromodynamics

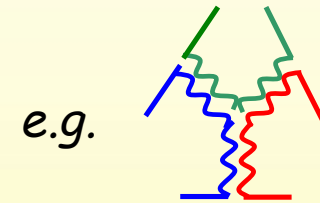
On the road to infrared slavery



Up, down quarks are relatively light, $m_{u,d} \sim 5 \text{ MeV}$, and thus relativistic



The interaction is a multi-gluon, and thus a multi-quark, process



Isospin symmetry is not obvious: $\varepsilon = \frac{m_d - m_u}{m_d + m_u} \sim \frac{1}{3}$



External probes can interact with collection of quarks

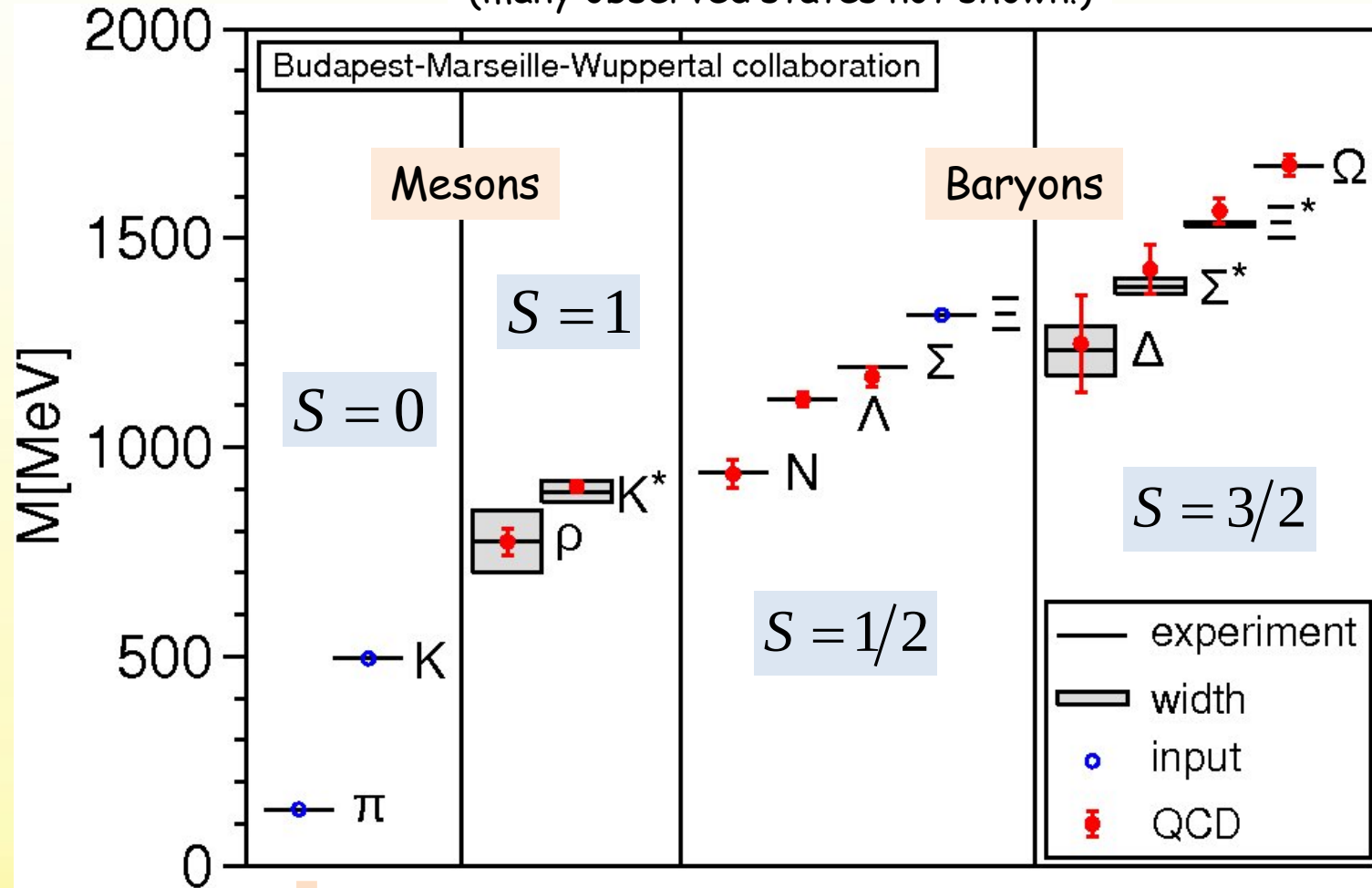
difficulty

quarks and gluons **not** the most convenient degrees of freedom at low energies

How does nuclear structure emerge from QCD?

Strongly interacting particles (hadrons)

(many observed states not shown!)



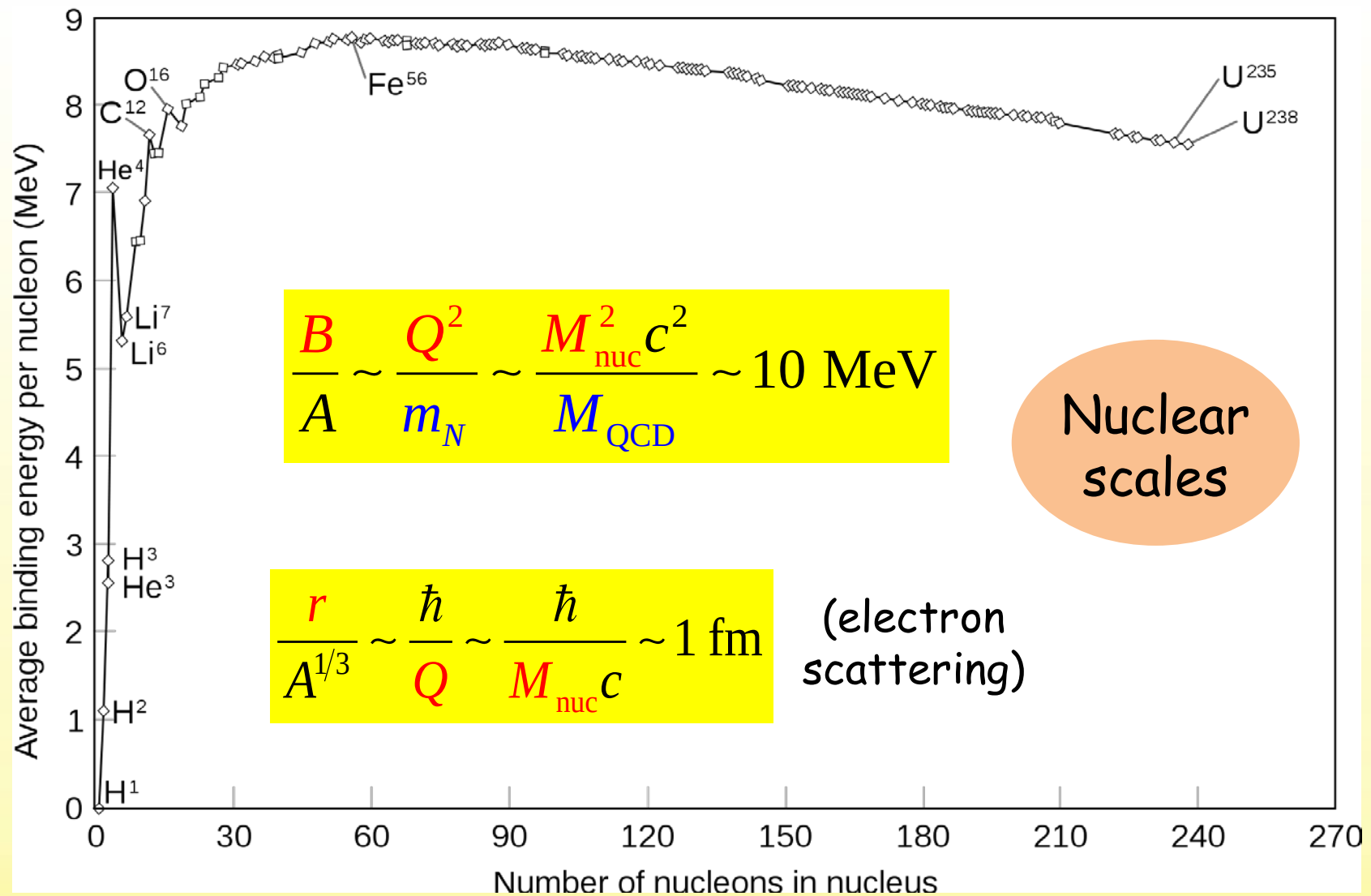
QCD
Scale

Exception: pion

$$m_{\pi} \approx 140 \text{ MeV}/c^2 \ll M_{\text{QCD}}$$

we'll return to it!

$$M_{\text{QCD}} \sim 1000 \text{ MeV}/c^2 = 1 \text{ GeV}/c^2$$



$$Q \sim M_{nuc} c \sim 100 \text{ MeV}/c$$

Multi-scale problems

H
atom

$$H = \left(\frac{p^2}{2m_e} - \frac{\alpha \hbar c}{r} \right) \left[1 + \mathcal{O} \left(\alpha; \frac{p^2}{m_e^2 c^2}; \frac{\hbar^2}{m_e^2 c^2 r^2} \right) \right] \quad \alpha \equiv \frac{e^2}{4\pi \hbar c} \cong \frac{1}{137} \ll 1$$

$$\left. \begin{array}{l} r \sim R \\ p \sim \frac{\hbar}{R} \end{array} \right\} E(R) \sim \left(\frac{\hbar^2}{2m_e R^2} - \frac{\alpha \hbar c}{R} \right)$$

$$\frac{dE(R)}{dR} = 0 \quad \Rightarrow \quad R = \frac{\hbar}{\alpha m_e c}$$

Three
scales

$$\left\{ \begin{array}{l} m_e c^2 = 0.5 \text{ MeV} \\ pc \sim \alpha m_e c^2 = 3.6 \text{ keV} \\ -E \sim \frac{p^2}{2m_e} \sim \frac{1}{2} \alpha^2 m_e c^2 = 13.6 \text{ eV} \end{array} \right. \quad \begin{array}{l} \alpha \\ \alpha \end{array}$$

(from now on, units such that $\hbar = 1, c = 1$)

However...

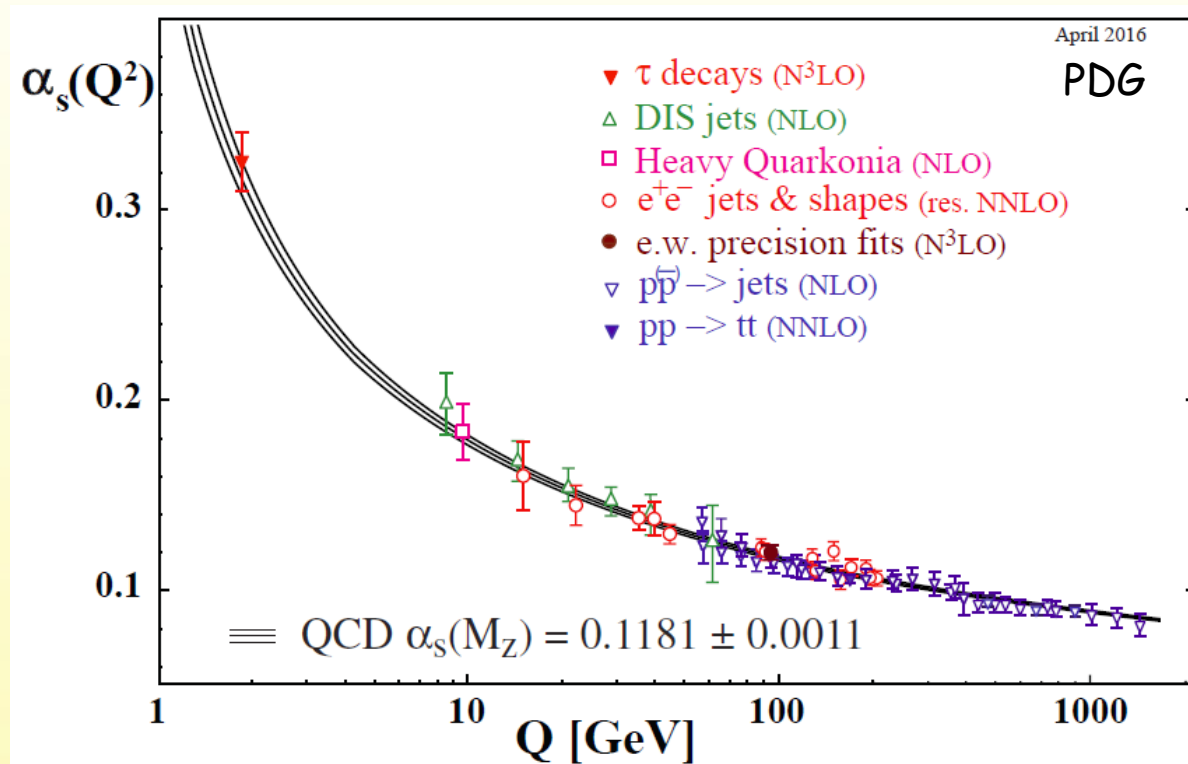
no obvious small coupling
in nuclear forces.

QCD
"fine-structure"
constant

Needed:

method that does not
rely on small couplings

~ 1



$\sim M_{\text{QCD}}$

➡ EFFECTIVE FIELD THEORY

“I do not believe that scientific progress is always best advanced by keeping an altogether open mind.

It is often necessary to forget one's doubts and to follow the consequences of one's assumptions wherever they may lead ---

the great thing is not to be free of theoretical prejudices, but to have the right theoretical prejudices.

And always, the test of any theoretical preconception is in where it leads.”

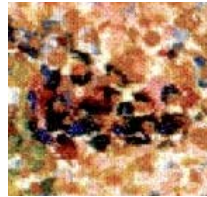
S. Weinberg

*The First Three Minutes,
1972*

Ingredients

- Relevant degrees of freedom





Ingredients

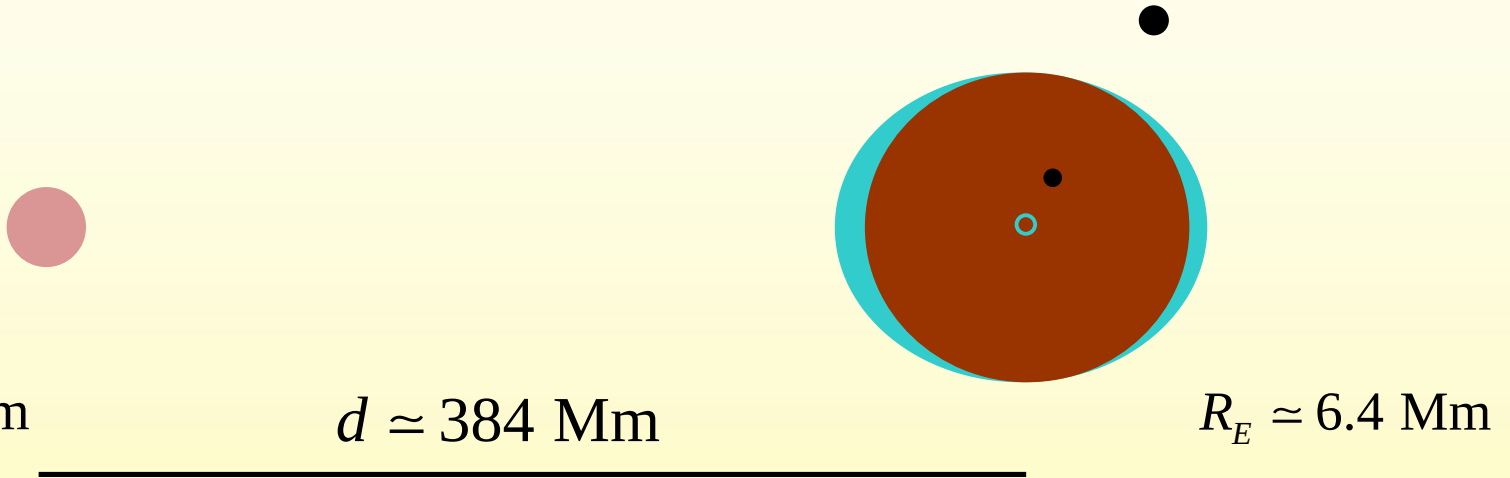
- Relevant degrees of freedom

choose the coordinates that fit the problem

- All possible interactions

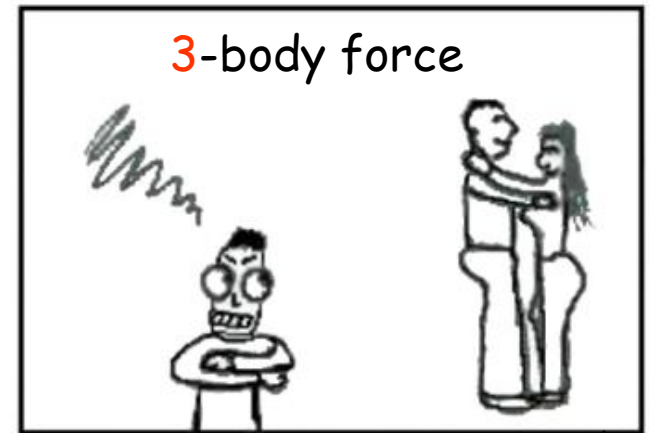


Example: Earth-moon-satellite system



2-body forces $\rightarrow$ 2+3-body forces

change in resolution



Ingredients

- Relevant degrees of freedom

choose the coordinates that fit the problem

- All possible interactions

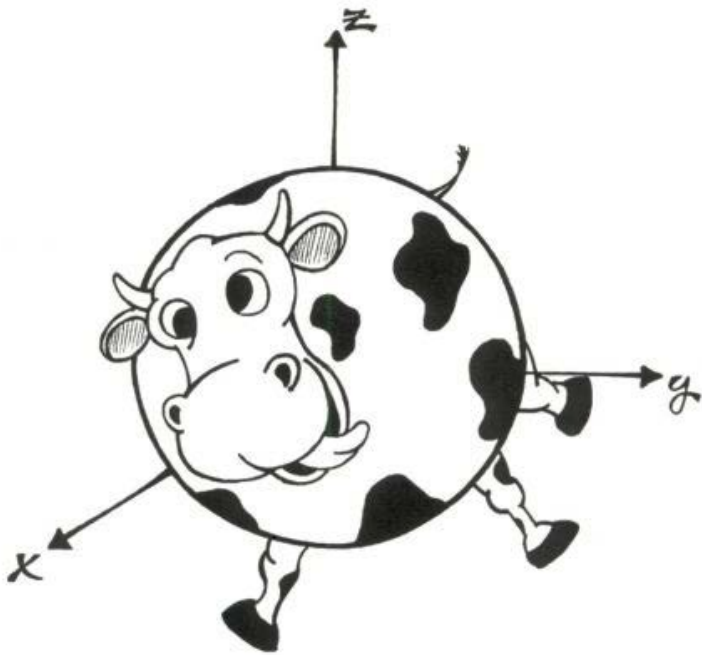
what is not forbidden is compulsory

- Symmetries



A farmer is having trouble with a cow whose milk has gone sour. He asks three scientists—a biologist, a chemist, and a physicist—to help him.

The biologist figures the cow must be sick or have some kind of infection, but none of the antibiotics he gives the cow work. Then, the chemist supposes that there must be a chemical imbalance affecting the production of milk, but none of the solutions he proposes do any good either. Finally, the physicist comes in and says,
"First, we assume a spherical cow..."



$$\sum_{ij} \alpha_{ij} u_i v_j \rightarrow \vec{u} \cdot \vec{v} + \sum_{ij} \delta \alpha_{ij} u_i v_j$$

no, say, $u_1 v_2$

$$|\delta \alpha_{ij}| \ll 1$$

amenable to
perturbation theory

Ingredients

- Relevant degrees of freedom

choose the coordinates that fit the problem

- All possible interactions

what is not forbidden is compulsory

- Symmetries

not everything is allowed

- Naturalness

After scales have been identified,
the remaining, dimensionless parameters are

$$\mathcal{O}(1)$$

unless suppressed by a symmetry

cow
non-sphericity...

Occam's razor:
simplest assumption, to be revised if necessary

fine-tuning

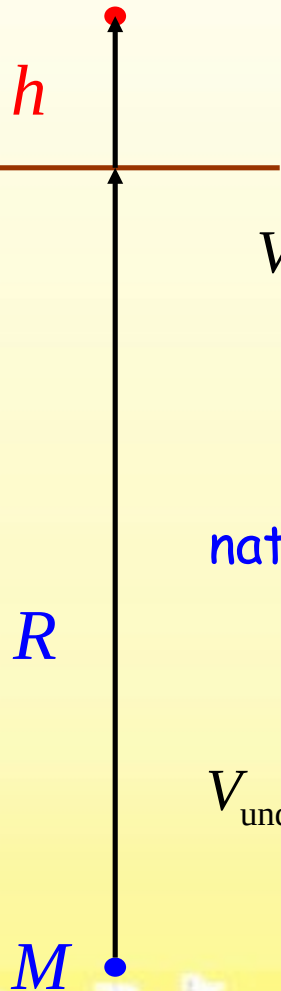
➡ Expansion in powers of

$$\frac{E}{E_{\text{und}}}$$

energy of probe

energy scale of
underlying theory

A classical example: the flat Earth light object near surface of a large body



$$E \sim mgh \ll E_{\text{und}} \equiv mgR \quad \left\{ \begin{array}{l} \text{d.o.f.: mass } m \\ \text{sym: } V_{\text{eff}}(h, x, y) = V_{\text{eff}}(h) \end{array} \right.$$

$$V_{\text{eff}}(h) = m \sum_{i=0}^{\infty} g_i h^i = \text{const} + mg \left\{ h + \frac{g_2}{g} h^2 + \dots \right\}$$

parameters

(neglecting quantum corrections...)

naturalness: $\frac{mg_{i+1}h^{i+1}}{mg_i h^i} = \frac{E}{E_{\text{und}}} \times \mathcal{O}(1) = \frac{h}{R} \times \mathcal{O}(1) \iff g_{i+1} = \mathcal{O}\left(\frac{g}{R^i}\right)$

$$V_{\text{und}}(h) = -GMm \frac{1}{R+h} = m \left(\frac{GM}{R^2} \right) \sum_{i=0}^{\infty} \left(\frac{-1}{R} \right)^{i-1} h^i \Rightarrow g_{i+1} = (-1)^i \frac{g}{R^i}$$

$h \ll R$

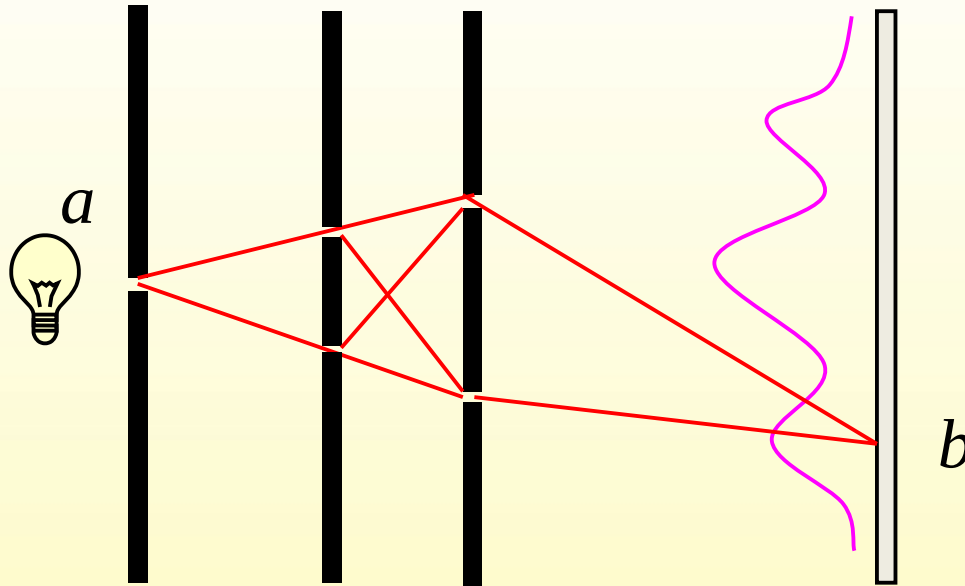
$\equiv g$

↕ 😊

itself the first term in a low-energy EFT of general relativity... 23

Going a bit deeper...

A short path to quantum mechanics



$$P = |A_1 + A_2 + A_3 + A_4|^2$$

sum over
all paths

$$A_i \propto \exp\left(i \int_a^b dt \mathcal{L}(q(t))\right)$$

each path contributes a phase
given by the classical* action

Path Integral

Feynman '48

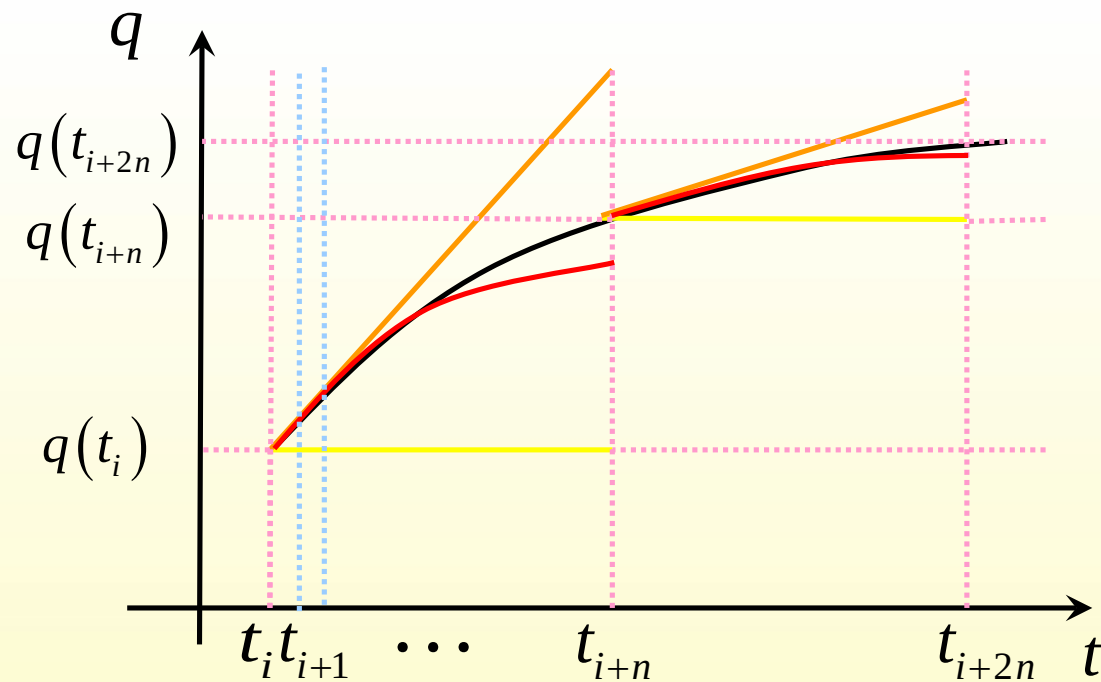
$$A = \int \mathcal{D}q \exp\left(i \int dt \mathcal{L}(q(t))\right)$$

$$\prod_i \int dq(t_i)$$

* Not strictly true when interactions involve
time derivatives (Salam + Strahdee '68)

classical
path

$$\delta\left(\int dt \mathcal{L}(q(t))\right) = 0$$



EFFECTIVE THEORY

$1/M$

← scale of fine-structure of dynamics

$1/Q$

← scale of variation of long-range dynamics

t_j t_{j+1}

t_{j+2}

← coarse-graining scale (cutoff)

$1/\Lambda$

$$\mathcal{L}(q(t_i)) \rightarrow \mathcal{L}\left(q(t_i) + \left.\frac{dq}{dt}\right|_{t_i} (t - t_i) + \frac{1}{2} \left.\frac{d^2q}{dt^2}\right|_{t_i} (t - t_i)^2 + \dots\right)$$

More generally,

$$\begin{aligned} A &= \int Dq \exp \left(i \int dt \mathcal{L}_{\text{und}}(q) \right) \\ &\quad \times \int D\tilde{q} \, \delta(\tilde{q} - f_{\Lambda}(q)) \quad \leftarrow \prod_i \int d\tilde{q}(t_i) \delta(\tilde{q}(t_i) - f(q(t_i))) \\ &= \int D\tilde{q} \exp \left(i \int dt \mathcal{L}_{\text{EFT}}(\tilde{q}) \right) \end{aligned}$$

$$\mathcal{L}_{\text{EFT}}(\tilde{q}) = \sum_{d,n=0}^{\infty} c_{d+n}(\mathbf{M}, \mathbf{\Lambda}) O_{d+n} \left(\tilde{q}, \left(\frac{d^d \tilde{q}}{dt^d} \right)^n \right)$$

Naturalness

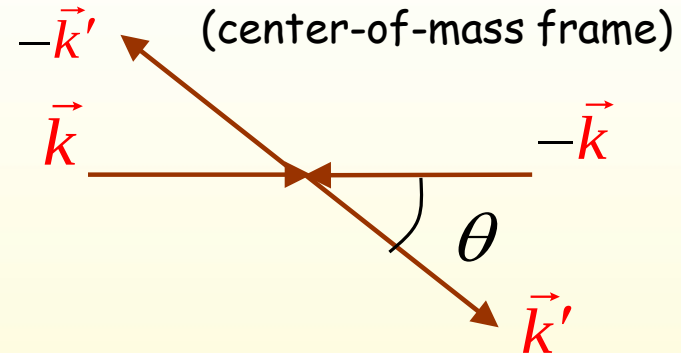
$$c_{d+n} \sim \frac{c_0}{M^{d+n}}$$

e.g. $V_{\text{EFT}}(\tilde{q}) = c_0 \tilde{q}^4 + c_2 \tilde{q}^2 \left(\frac{d\tilde{q}}{dt} \right)^2 + \dots$

Observables $\sim$ expansion in $\frac{Q}{M}$

All information is in the S matrix...

elastic scattering
(for simplicity)



$$\left\{ \begin{array}{l} |\vec{k}'| = |\vec{k}| \equiv k \quad (\text{conservation of energy}) \end{array} \right.$$

θ : given by certain probability amplitude -
the "scattering amplitude"

$$T(k, \theta) = \sum_{l=0}^{\infty} T_l(k) P_l(\cos \theta)$$

angular momentum partial-wave amplitude

Legendre polynomial

1. Parametrization by phase shifts: $T_l(k) = -\frac{4\pi i}{\mu k} [\exp(2i\delta_l(k)) - 1]$

2. Low-energy expansion: $T_l^{-1}(k) = \frac{\mu}{2\pi k^{2l}} \left[-\frac{1}{a_l} + \frac{r_l}{2} k^2 + \frac{P_l}{4} k^4 + \dots - i k^{2l+1} \right]$

effective range shape parameter

scattering length

3. Poles: $T_l^{-1}(K_r + iK_i) = 0$ $\left\{ \begin{array}{l} K_r = 0 \text{ bound states} \Rightarrow E = -B < 0 \\ K_i \leq 0 \text{ resonances} \Rightarrow E = E_R - i\Gamma_R/2 \end{array} \right.$

characteristic external momentum

$$\left\{ \begin{array}{l} T = T^{(\infty)}(\textcolor{red}{Q}) \sim \underbrace{N(\textcolor{blue}{M})}_{\text{normalization}} \sum_{\nu=\nu_{\min}}^{\infty} \sum_i \left[\frac{\textcolor{red}{Q}}{\textcolor{blue}{M}} \right]^{\nu} F_{\nu} \left(\underbrace{\frac{\textcolor{red}{Q}}{\textcolor{red}{m}}, \frac{\textcolor{red}{Q}}{\textcolor{blue}{\Lambda}}}_{\text{"non-analytic", from the solution of a dynamical equation (e.g. Schrödinger eq.)}}, \underbrace{\{c_{\nu}(\textcolor{blue}{\Lambda})\}}_{\text{"low-energy constants" "Wilson coefficients" PCKLJS}} \right) \\ \frac{dT}{d\textcolor{blue}{\Lambda}} = 0 \quad \text{"renormalization-group invariance"} \end{array} \right.$$

$$\nu = \nu(d, n, \dots) \quad \text{"power counting"}$$

For $\textcolor{red}{k} \sim \textcolor{red}{m}$, truncate consistently with RG invariance so as to allow systematic improvement (perturbation theory):

$$T = T^{(\bar{\nu})} \left[1 + \mathcal{O} \left(\frac{\textcolor{red}{Q}}{\textcolor{blue}{M}}, \frac{\textcolor{red}{Q}}{\textcolor{blue}{\Lambda}} \right) \right] \quad \frac{\textcolor{blue}{\Lambda}}{T^{(\bar{\nu})}} \frac{dT^{(\bar{\nu})}}{d\textcolor{blue}{\Lambda}} = \mathcal{O} \left(\frac{\textcolor{red}{Q}}{\textcolor{blue}{\Lambda}} \right)$$

† Warning: renormalization **NOT** optional †
Otherwise, just a model since dependent on regularization choice

“second quantization”:

+ Lorentz invariance

$$q(t) \rightarrow \psi(\vec{r}, t), \psi^*(\vec{r}, t)$$

representation of $SO(3,1)$

$$dt \rightarrow dt d^3r$$

$$\equiv d^4x$$

$$\frac{d}{dt} \rightarrow \frac{\partial}{\partial t}, \frac{\partial}{\partial \vec{r}}$$

$$\rightarrow \frac{\partial}{\partial x^\mu}$$

EFFECTIVE FIELD THEORIES

Euler + Heisenberg '36

Weinberg '67 ... '79

Wilson, early 70s

...

$$\begin{aligned}
 A &= \int D\psi D\psi^* \exp\left(i \int d^4x \left\{ \mathcal{L}_{\text{free}}(\psi) + \mathcal{L}_{\text{int}}(\psi) \right\}\right) \\
 &= \int D\psi D\psi^* \left\{ 1 + i \int d^4x \mathcal{L}_{\text{int}}(\psi) + \frac{1}{2} \left[i \int d^4x \mathcal{L}_{\text{int}}(\psi) \right]^2 + \dots \right\} \exp\left(i \int d^4x \mathcal{L}_{\text{free}}(\psi)\right)
 \end{aligned}$$

momentum space

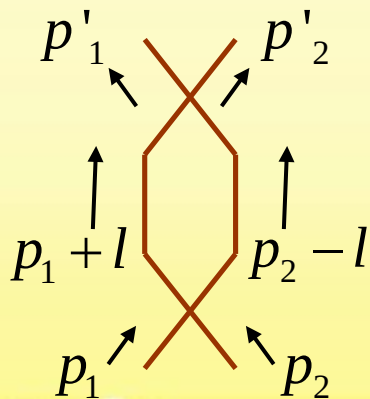


(skip many steps...)



$$\mathcal{L}_{\text{int}} = \frac{\lambda}{4} (\psi^* \psi)^2 \quad \times \quad = i\lambda$$

$$\left| \quad = \frac{i}{p^2 - m^2 + i\varepsilon} \right.$$



$$= \int \frac{d^4l}{(2\pi)^4} i\lambda \frac{i}{(p_1 + l)^2 - m^2 + i\varepsilon} \frac{i}{(p_2 - l)^2 - m^2 + i\varepsilon} i\lambda$$

= ...

but divergent from high-momentum region...



needs a cutoff to separate high and low momenta

EFFECTIVE FIELD THEORIES

Euler + Heisenberg '36
Weinberg '67 ... '79
Wilson, early 70s
...

Two possibilities:

- know and can solve underlying theory --
get c_i 's in terms of parameters in $\mathcal{L}_{\text{und}}$
- know but cannot solve, or do not know, underlying theory --
invoke Weinberg's "folk theorem":

Weinberg '79

"The quantum field theory generated by the most general Lagrangian with some assumed symmetries will produce the most general S matrix incorporating quantum mechanics, Lorentz invariance, unitarity, cluster decomposition and those symmetries, with no further physical content."

Note: proven only for scalar field with Z_2 symmetry in E_4 , Ball + Thorne '94
but no known counterexamples

Bira's EFT Recipe

1. identify degrees of freedom and symmetries
2. construct most general Lagrangian
3. run the methods of field theory

what is not forbidden
is mandatory!

- compute Feynman diagrams with all momenta $Q < \Lambda$
("regularization")
- relate $c_i(\Lambda), \Lambda$ to observables, which should be independent of Λ
("renormalization")

not a model form factor!!!

➡ controlled expansion in $\frac{Q}{M} \times \mathcal{O}(1)$
"naturalness": what else?
unless suppressed by symmetry...

contrast to models, which have fewer, but *ad hoc*, interactions;
useful in the identification of relevant degrees of freedom
and symmetries, but plagued with uncontrolled errors

“Nonrenormalizable theories
are as renormalizable
as renormalizable theories.”

S. Weinberg



A modern view of renormalization...

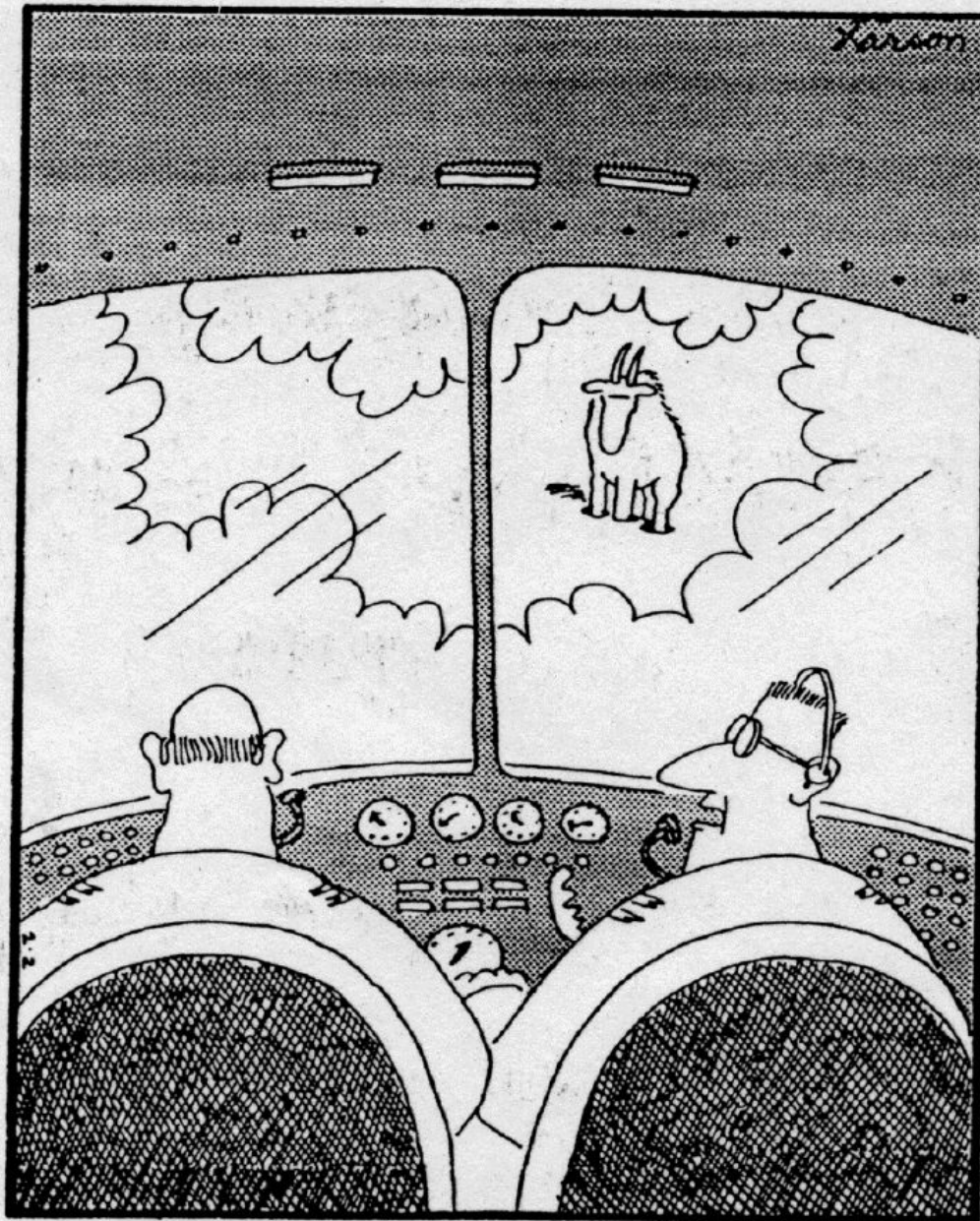
“A significant change in physicists’ attitude towards what should be taken as a guiding principle in theory construction is taking place in recent years in the context of the development of EFT.

For many years (...) renormalizability has been taken as a necessary requirement.

Now, considering the fact that experiments can probe only a limited range of energies, it seems natural to take EFT as a general framework for analyzing experimental results.”

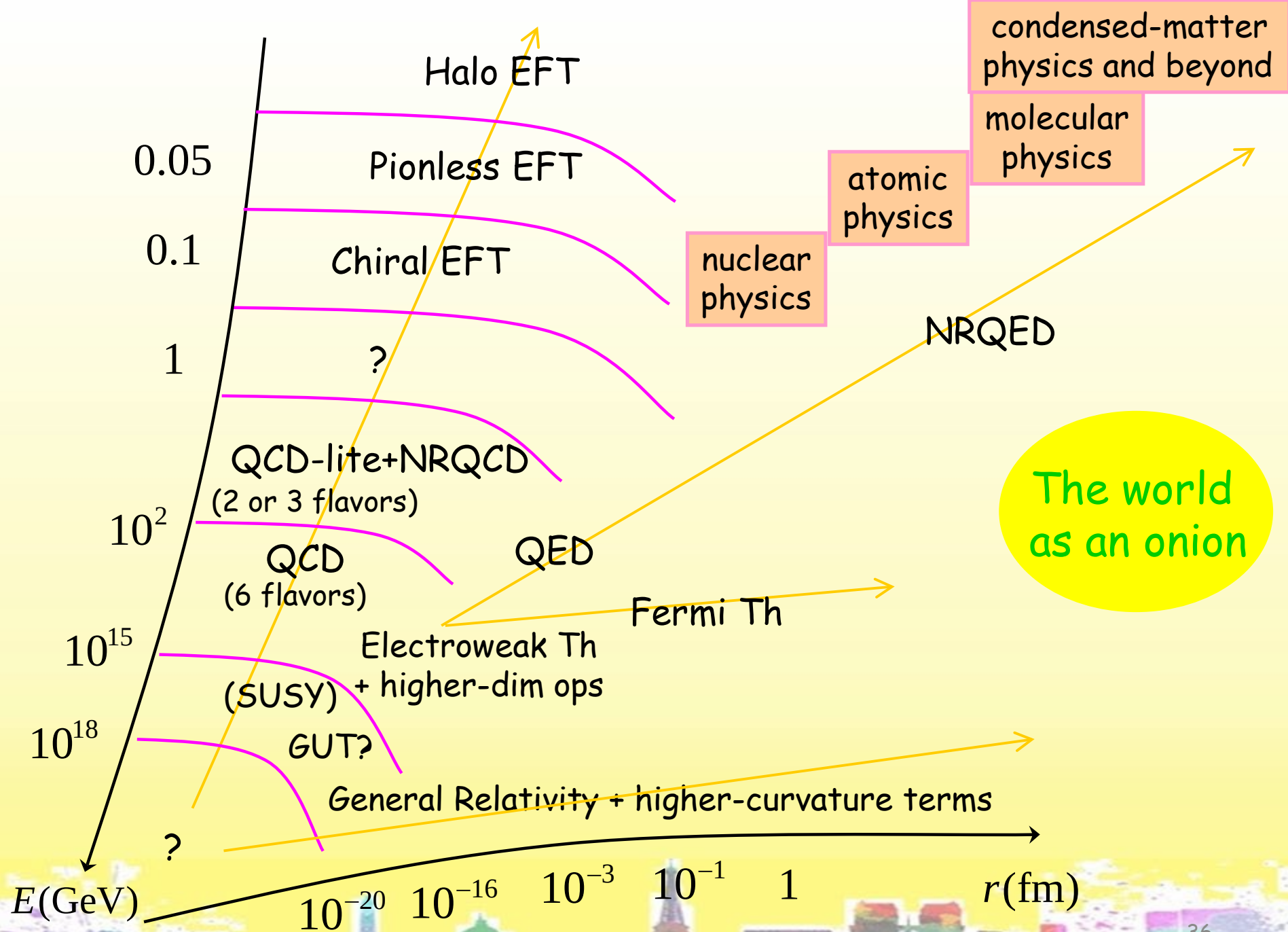
T. Y. Cao

in *Renormalization, From Lorentz to Landau (and Beyond)*,
L.M. Brown (ed.), 1993



Time for a
paradigm
change,
perhaps?

"Say . . . What's a mountain goat doing way
up here in a cloud bank?"



Summary

- ◆ Nuclear systems involve multiple scales but **no** obvious small coupling constant
- ◆ EFT is a **general** framework to deal with a multi-scale problem using the **small** ratio of scales as an expansion parameter
- ◆ EFT is **not** just any field theory with “effective” degrees of freedom
- ◆ Symmetries and renormalization **essential** for model independence

Stay tuned:
next, how we can describe low-energy nuclear physics systematically

INTRODUCTION TO NUCLEAR EFFECTIVE FIELD THEORIES[©]

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Supported in part by
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O u t l i n e

- Effective Field Theories
- EFT for Short-Range Forces
 - ▶ Scales
 - ▶ Degrees of freedom
 - ▶ Symmetries
 - ▶ Action
- Two-Body System
- Three-Body System
- More-Body Systems
- Long-range Forces

Very Brief History of Nuclear EFTs

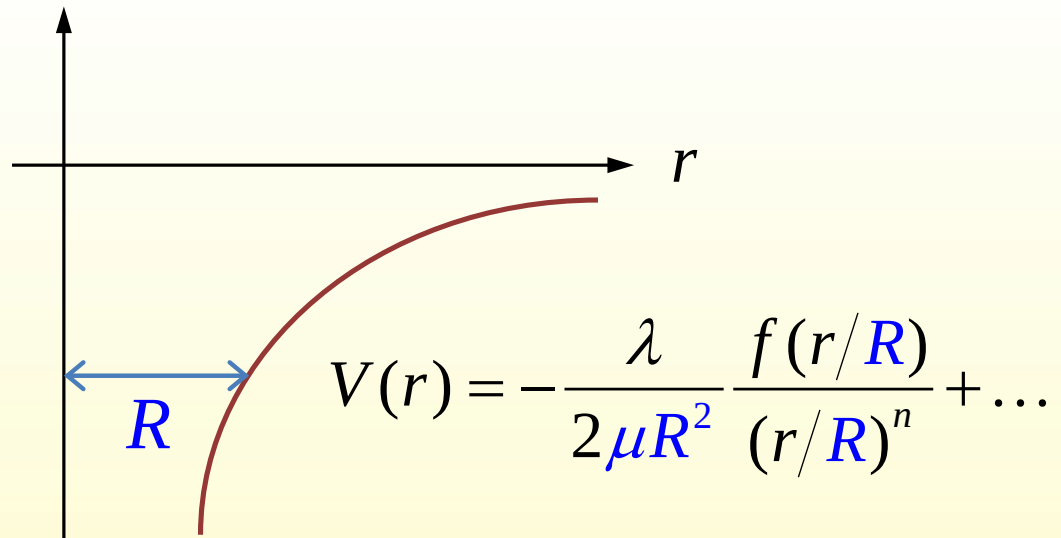
- ≥ 1990 : Weinberg generalizes Chiral Perturbation Theory to nuclei
--- "Chiral EFT". He identifies infrared enhancements in multi-nucleon amplitudes and suggests:
 - i) the potential be calculated in the usual ChPT expansion;
 - ii) amplitudes be calculated from the exact solution of the Schrödinger equation with truncated potentials
- ≥ 1992 : Others carry out this procedure obtaining good description of few-body data with only limited cutoff variation
- ≥ 1996 : Weinberg's prescription shown to fail to produce renormalizable amplitudes; problem ignored by most nuclear physicists
- ≥ 1998 : Pionless EFT developed to resolve these issues; it additionally finds great success for light nuclei
- ≥ 1998 : Pionless EFT extended with perturbative pions and shown to work more or less as well as Pionless EFT
- ≥ 2005 : Weinberg's power counting modified so that nonperturbative pions produce renormalizable amplitudes
- today: Phenomenological problems with Weinberg's prescription mounting in the case of heavier nuclei

$$B_3 = 8.48 \text{ MeV}$$

$3B_A/(AB_3)$	nucleons	
A	experiment	
2	0.39	
3	1	
3*	(0.17)	
4	2.50	
4*	0.71	
5	1.94	
6	1.89	
$\vdots$		
16	2.82	
$\vdots$		
$\rightarrow \infty$	5.7	

$= \kappa \left(1 + \eta A^{-1/3} + \dots \right)$ "liquid-drop formula" v. Weizsäcker '36

Finite-range two-body potential



nucleons:
pion
exchange

$$\left\{ \begin{array}{l} 2\mu = m_N \simeq 940 \text{ MeV} \\ n = 3, \dots \\ R = 1/m_\pi \simeq 1.4 \text{ fm} \\ \lambda = m_\pi / M_{NN} \simeq 1/2 \\ f(r/R) = \exp(-r/R) \end{array} \right.$$

^4He atoms:
two-photon
exchange

$$\left\{ \begin{array}{l} 2\mu = m_{^4\text{He}} \\ n = 6, \dots \\ R = l_{\text{vdW}} \simeq 5.4 \text{ \AA} \\ \lambda = 1 \\ f(r/R) = 1 \end{array} \right.$$

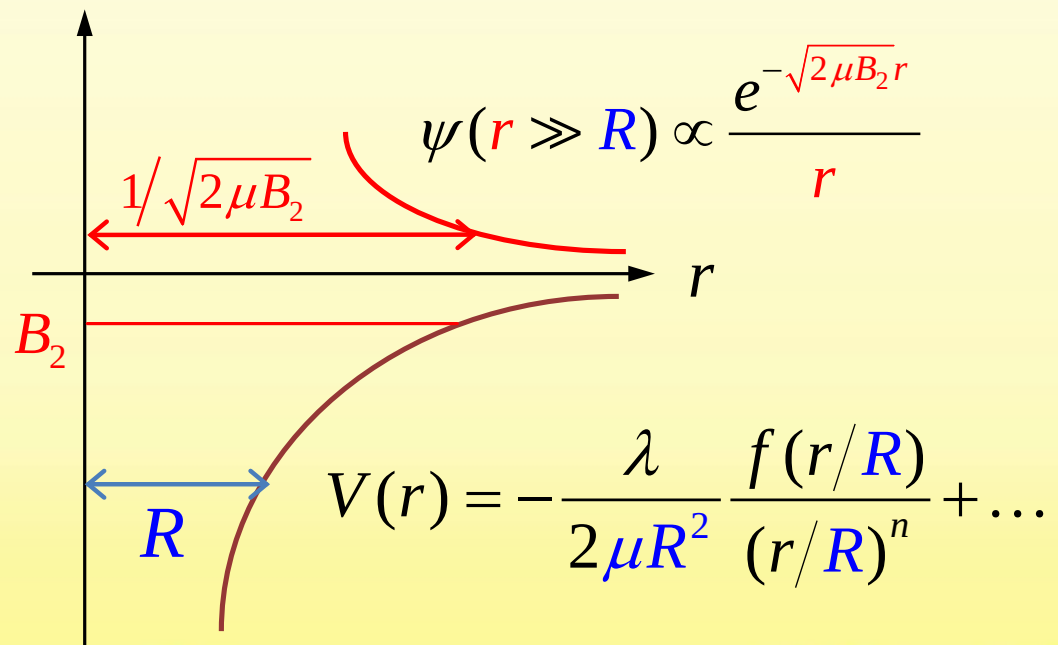
A measure of (inverse) size: the binding momentum

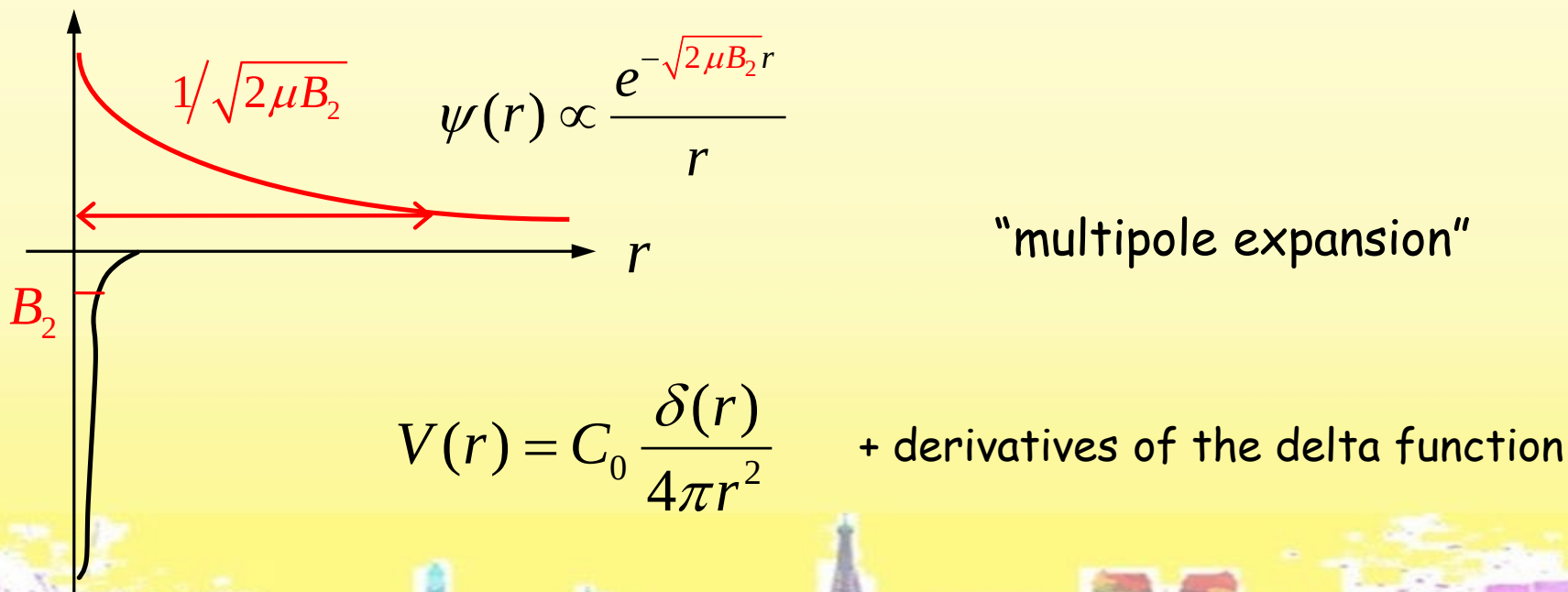
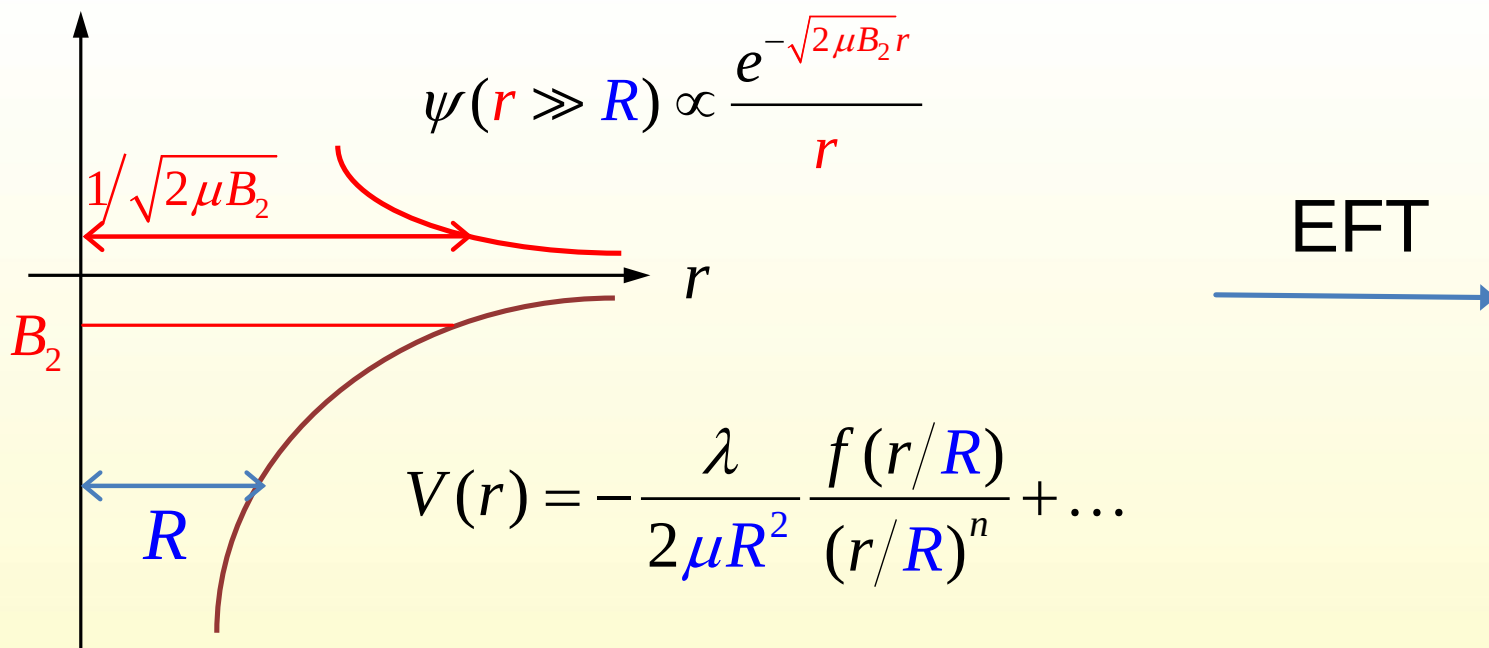
$$Q_A \equiv \sqrt{2m B_A / A} \quad \left\{ \begin{array}{ll} Q_2 = \sqrt{2\mu B_2} & \text{right position of two-body pole} \\ B_{A \gg 1} \sim \sum_{i=1}^{A \gg 1} \frac{Q_i^2}{2m} & \text{all particles contribute equally to energy} \end{array} \right.$$

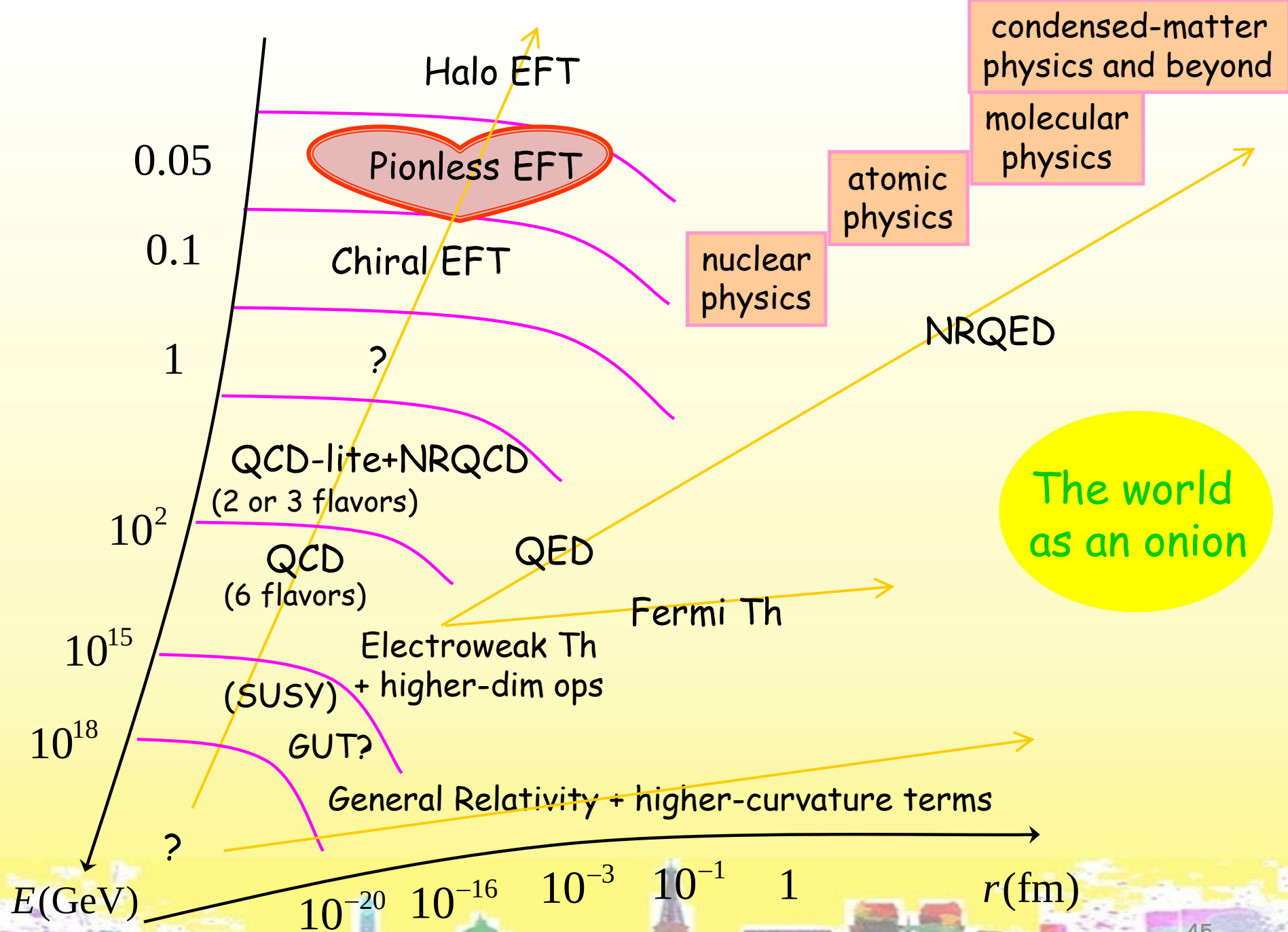
nucleons: $(Q_3 R)^{-1} \simeq 2$
 $(Q_2 R)^{-1} \simeq 3$

^4He atoms: $(Q_3 R)^{-1} \simeq 2$
 $(Q_2 R)^{-1} \simeq 10$

"unitarity limit" $(Q_2 R)^{-1} \rightarrow \infty$







On blackboard



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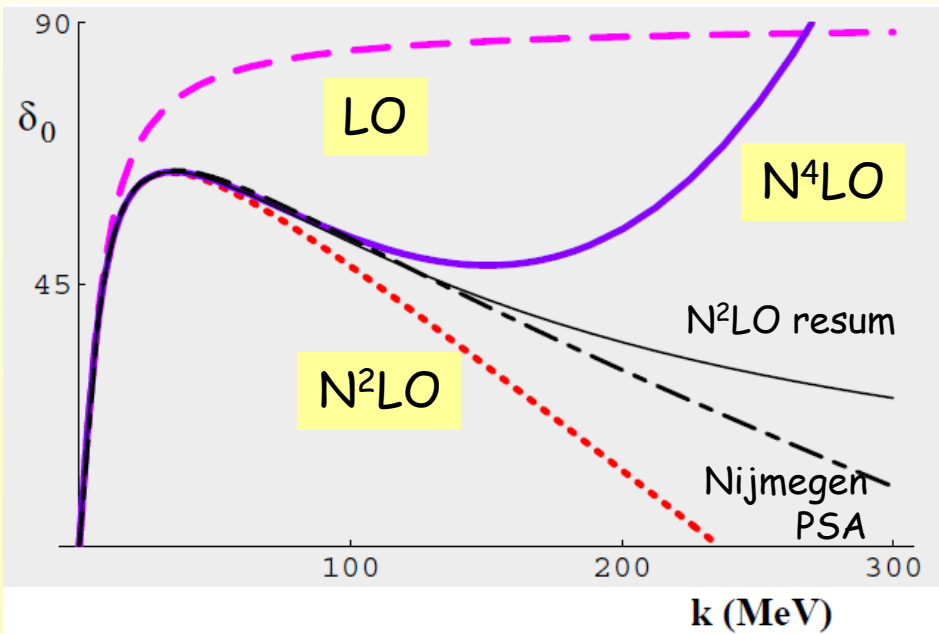
- Effective Field Theories
- EFT for Short-Range Forces
- Two-Body System
 - ▶ Amplitude
 - ▶ Connection to ERE, potentials
 - ▶ Potentials and fine tuning
 - ▶ Non-trivial fixed point, scale invariance
- Three-Body System
- More-Body Systems
- Long-range Forces

On blackboard



Nucleon-nucleon phase shifts

Pionless EFT

 $^1S_0$ 

fit

$$C_{01}^{(0)} \Rightarrow a_0 = -20.0 \text{ fm (exp)}$$

$$C_{21}^{(1)} \Rightarrow r_0 = 2.78 \text{ fm (exp)}$$

predict

$$B_{d^*} = 0.09 \text{ MeV (NLO)}$$

 $^3S_1$

fit

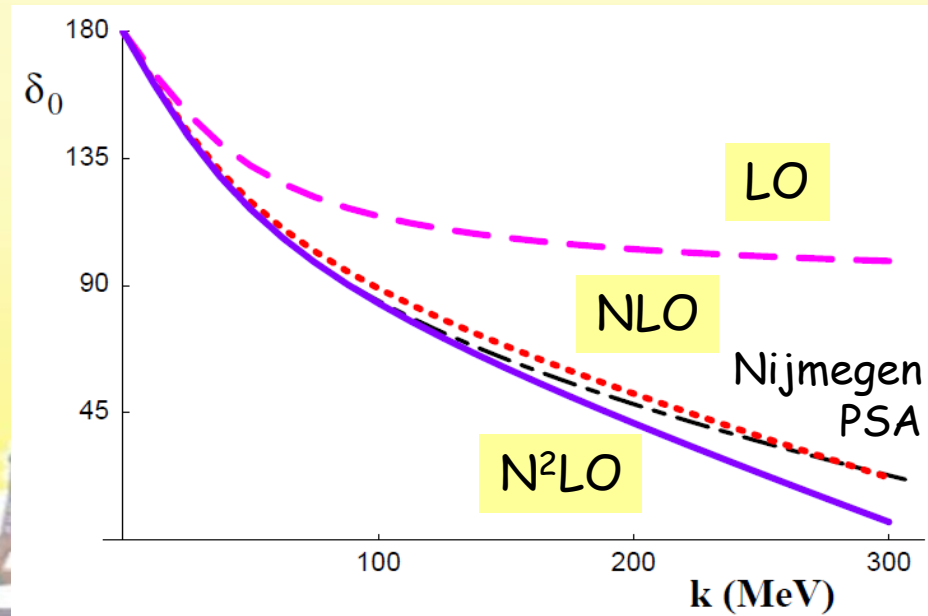
$$C_{00}^{(0)} \Rightarrow a_1 = 5.42 \text{ fm (exp)}$$

$$C_{20}^{(1)} \Rightarrow r_1 = 1.75 \text{ fm (exp)}$$

predict

$$B_d = 1.91 \text{ MeV (NLO)}$$

$$B_d = 2.22 \text{ MeV (exp)}$$



INTRODUCTION TO NUCLEAR EFFECTIVE FIELD THEORIES[©]

U. van Kolck

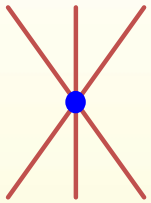
*Institut de Physique Nucléaire d'Orsay
and University of Arizona*

Supported in part by
CNRS and US DOE

O
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- Effective Field Theories
- EFT for Short-Range Forces
- Two-Body System
- Three-Body System
 - ▶ Auxiliary Field
 - ▶ Amplitude
 - ▶ Bound states and correlations
 - ▶ Limit cycle, discrete scale invariance
- More-Body Systems
- Long-range Forces

Issue: at what order a three-body force?

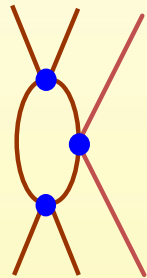


$$\sim i \frac{(4\pi)^2}{m} D_0(\Lambda)$$

$$\text{NDA: } D_0 \sim \frac{1}{M_{hi}^4}$$

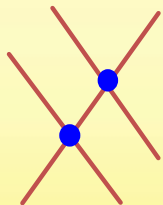
but with fine tuning?

First, LO two-body force:



$$\sim \left(\frac{4\pi C_0}{m} \right)^3 \frac{Q^3}{4\pi} \left(\frac{m}{Q^2} \right)^2$$

$$\sim \frac{4\pi C_0}{m} \frac{mQ}{4\pi} \sim \frac{Q}{M_{lo}}$$



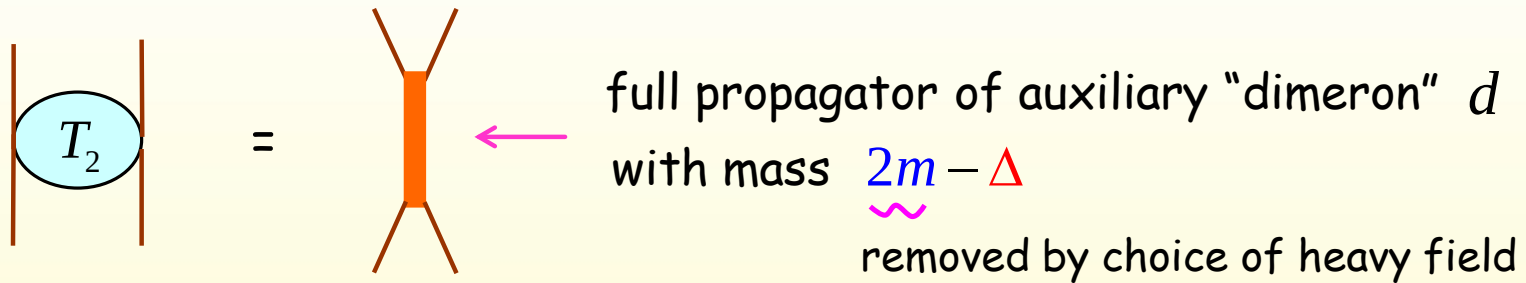
$$\sim \left(\frac{4\pi C_0}{m} \right)^2 \frac{m}{Q^2}$$

if there is a shallow two-body state



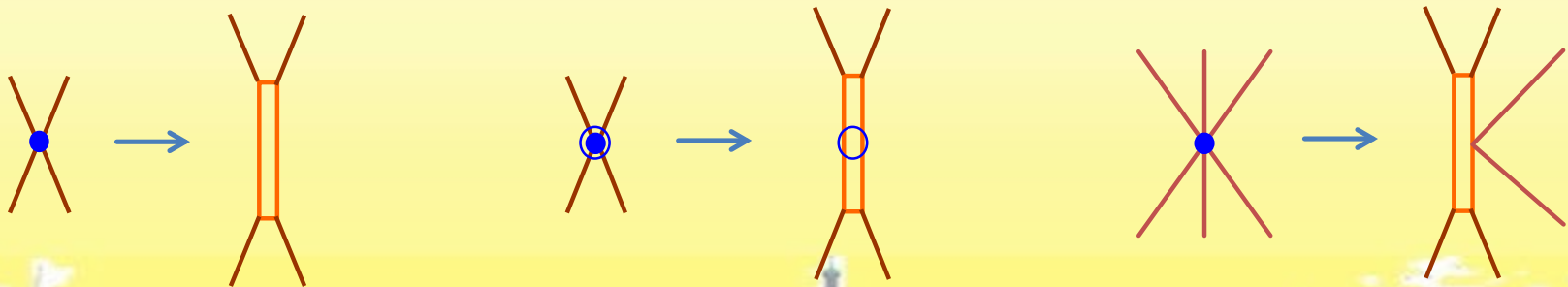
need to resum LO two-body force

Auxiliary field



$$\mathcal{L} \rightarrow \mathcal{L}_d = \psi^\dagger \left(i \frac{\partial}{\partial t} + \frac{\nabla^2}{2m} + \dots \right) \psi + d^\dagger \left[\overset{\text{LO LEC}}{\Delta} + \overset{\text{NLO LEC}}{\sigma} \left(i \frac{\partial}{\partial t} + \frac{\nabla^2}{4m} \right) + \dots \right] d$$

$$- \sqrt{\frac{2\pi}{m}} \left[d^\dagger \psi \psi + \psi^\dagger \psi^\dagger d \right] - h_0 d^\dagger d \psi^\dagger \psi + \dots$$



$$\text{[Diagram: vertical rectangle]} \sim \frac{i}{\Delta}$$

$$\text{[Diagram: Y-junction]} \sim i\sqrt{\frac{2\pi}{m}}$$

$$\text{[Diagram: inverted Y-junction]} \sim i\sqrt{\frac{2\pi}{m}}$$

$$\begin{aligned} \text{[Diagram: vertical rectangle with a bubble]} &\sim -2 \frac{2\pi}{m} \left(\frac{i}{\Delta} \right)^2 \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^0 + p^0 - \frac{(\vec{l} + \vec{p})^2}{2m_N} + i\varepsilon} \frac{1}{-l^0 + p^0 - \frac{(\vec{l} + \vec{p})^2}{2m_N} + i\varepsilon} \\ &= \dots = -\frac{i}{\Delta^2} \left(\theta_1 \Lambda + i k + \theta_{-1} \frac{k^2}{\Lambda} + \dots \right) \end{aligned}$$

LO

$$\begin{aligned} \text{[Diagram: thick vertical bar]} &= \text{[Diagram: vertical rectangle]} + \text{[Diagram: vertical rectangle with a bubble]} + \dots \\ &= \dots = i \underbrace{(\Delta + \theta_1 \Lambda + i k)}_{\equiv \Delta_R = 1/a_2}^{-1} + \mathcal{O}\left(\frac{1}{M_{hi}}\right) \end{aligned}$$

NLO

$$\begin{aligned} \text{[Diagram: thick vertical bar with a circle]} &= \dots = -i \left(\Delta_R + i k \right)^{-2} \underbrace{\left(\sigma + \theta_{-1} \frac{m}{\Lambda} \right)}_{\equiv \sigma_R = -m r_2 / 2} \frac{k^2}{m} \end{aligned}$$

renormalization simpler
than before because
momentum $\rightarrow$ energy

$$\begin{aligned}
 & \frac{k^2}{4m} - B_2 + E', \vec{p}' \quad \frac{k^2}{2m} - E', -\vec{p}' \\
 & \text{LO} \quad T_3 = \text{diagram 1} + \text{diagram 2} + \dots \\
 & \frac{k^2}{2m}, -\vec{p} \quad \frac{k^2}{4m} - B_2, \vec{p} \quad \sim \frac{4\pi}{Q^2} \quad \sim \frac{Q^3}{4\pi} \left(\frac{4\pi}{Q^2} \right)^2 \frac{1}{\Delta} \sim \frac{4\pi}{Q^2} \frac{Q}{M_{lo}} \\
 & E = \frac{3k^2}{4m} - B_2 \\
 & = \text{diagram 3} + \text{diagram 4}
 \end{aligned}$$

$$T_3 = K_{OPE} + \lambda \int_0^\Lambda \frac{d^3 l}{(2\pi)^3} \frac{T_3 K_{OPE}}{D}$$

Skorniakov--Ter-Martirosian '57

Only “half-off-shell” amplitude enters: $E' = \frac{k^2 - \vec{p}'^2}{2m}$

$$T_3(\vec{p}', \vec{p}) = -v_3(\vec{p}', \vec{p}) - \lambda \int_0^\Lambda \frac{d^3l}{(2\pi)^3} \frac{T_3(\vec{p}', \vec{l}) v_3(\vec{l}, \vec{p})}{-1/a_2 + \sqrt{3l^2/4 - mE}}$$

$$v_3(\vec{p}', \vec{p}) = \frac{8\pi}{mE - \vec{p}'^2 - \vec{p}^2 - \vec{p}' \cdot \vec{p}}$$

bosons $S = 0$ $\lambda = 1$

nucleons $\left\{ \begin{array}{ll} S = 3/2 & \lambda = -1/2 \\ S = 1/2 & \text{two equations that decouple in the UV} \end{array} \right. \quad \begin{array}{l} \lambda = 1 \text{ \&} \\ \lambda = -1/2 \end{array}$

Simplest case: S wave

$$\begin{aligned}
 T_{3,0}(p', k) &= -v_{3,0}(p', k) - \frac{\lambda}{2\pi^2} \int_0^\Lambda dl l^2 \frac{T_{3,0}(p', l) v_{3,0}(l, k)}{-1/a_2 + \sqrt{3l^2/4 - mE}} \\
 &= -v_{3,0}(p', k) - \frac{\lambda}{2\pi^2} \int_0^\Lambda dl l^2 \frac{v_{3,0}(p', l) T_{3,0}(l, k)}{-1/a_2 + \sqrt{3l^2/4 - mE}} \\
 v_{3,0}(p', k) &= \frac{4\pi}{p'k} \ln \left(\frac{p'^2 + k^2 - p'k - mE}{p'^2 + k^2 + p'k - mE} \right)
 \end{aligned}$$

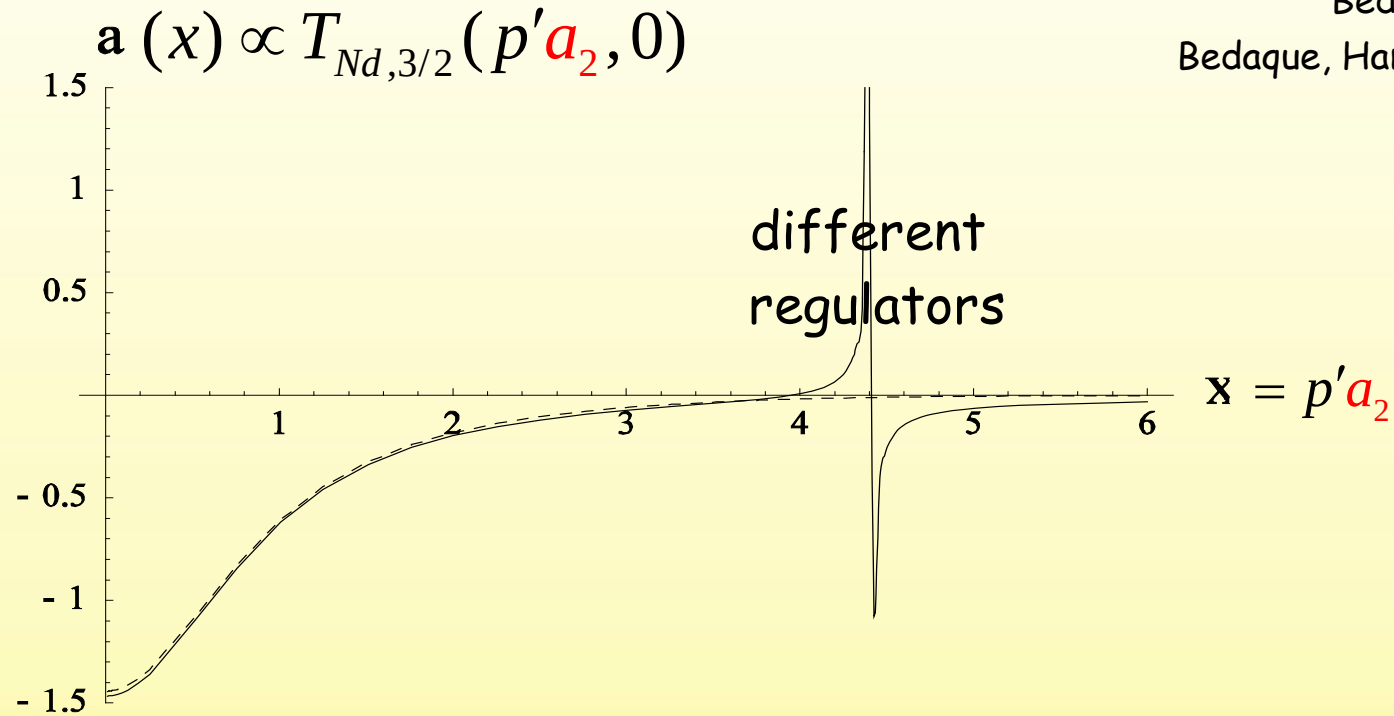
As before, first look at UV properties: $p' \gg k \gtrsim 1/a_2$

$$T_{3,0}(p' \gg k) = \frac{4\lambda}{\sqrt{3}\pi} \int \frac{dl}{p'} \ln \left(\frac{p'^2 + p'l + l^2}{p'^2 - p'l + l^2} \right) T_{3,0}(l \gg k)$$

$$\text{Ansatz } T_{3,0}(p' \gg k) \propto p'^{-(s+1)} \Rightarrow \frac{8\lambda}{\sqrt{3}s} \frac{\sin(s\pi/6)}{\cos(s\pi/2)} = 1$$

Danilov '63

nucleons $S = 3/2$ $\lambda = -1/2 \Rightarrow s \simeq 2.17$



Bedaque + v.K. '97
Bedaque, Hammer + v.K. '98
...

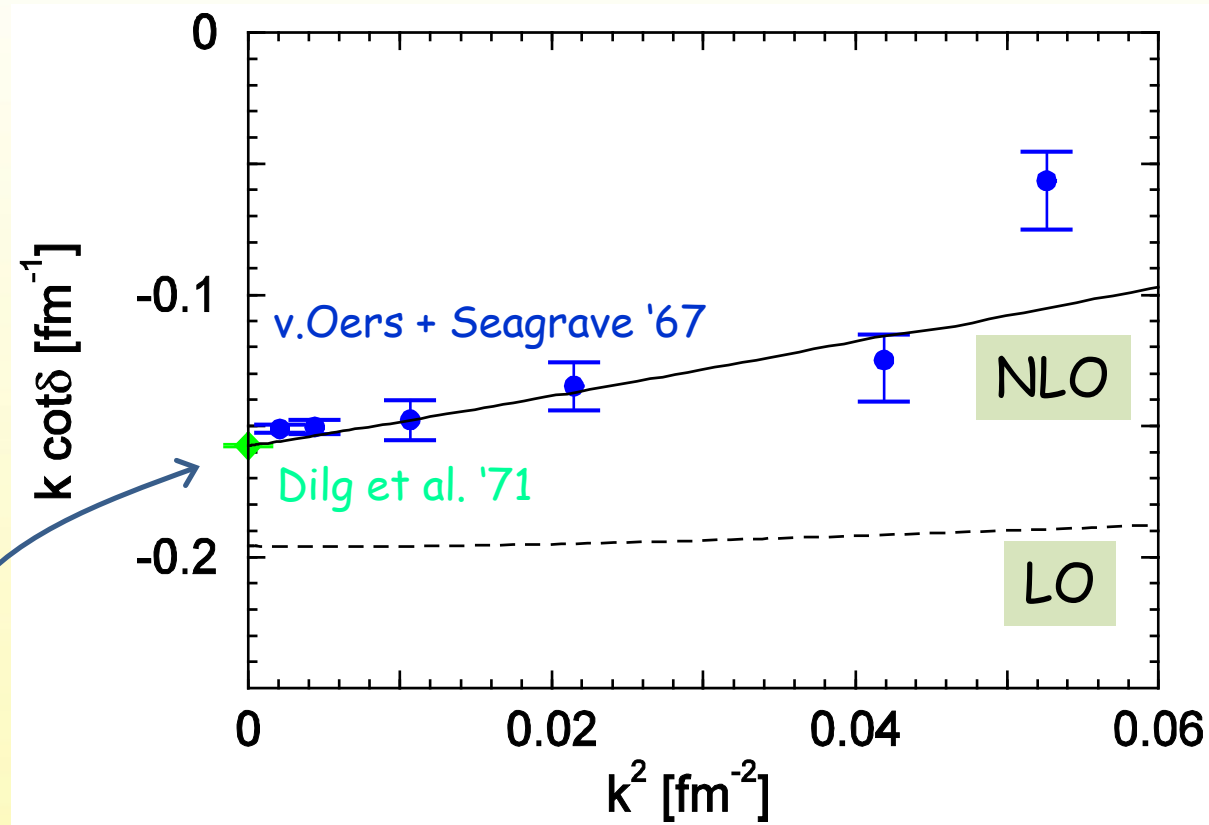
$$\frac{\partial T_{Nd,3/2}(p'a_2 \sim 1, 0)}{\partial \Lambda} \cong 0$$

$$T_{Nd,3/2}(p'a_2 \gg 1, 0) \propto p'^{-3.17}$$

nucleons $S = 3/2$

Bedaque + v.K. '97
Bedaque, Hammer + v.K. '98
...

postdict



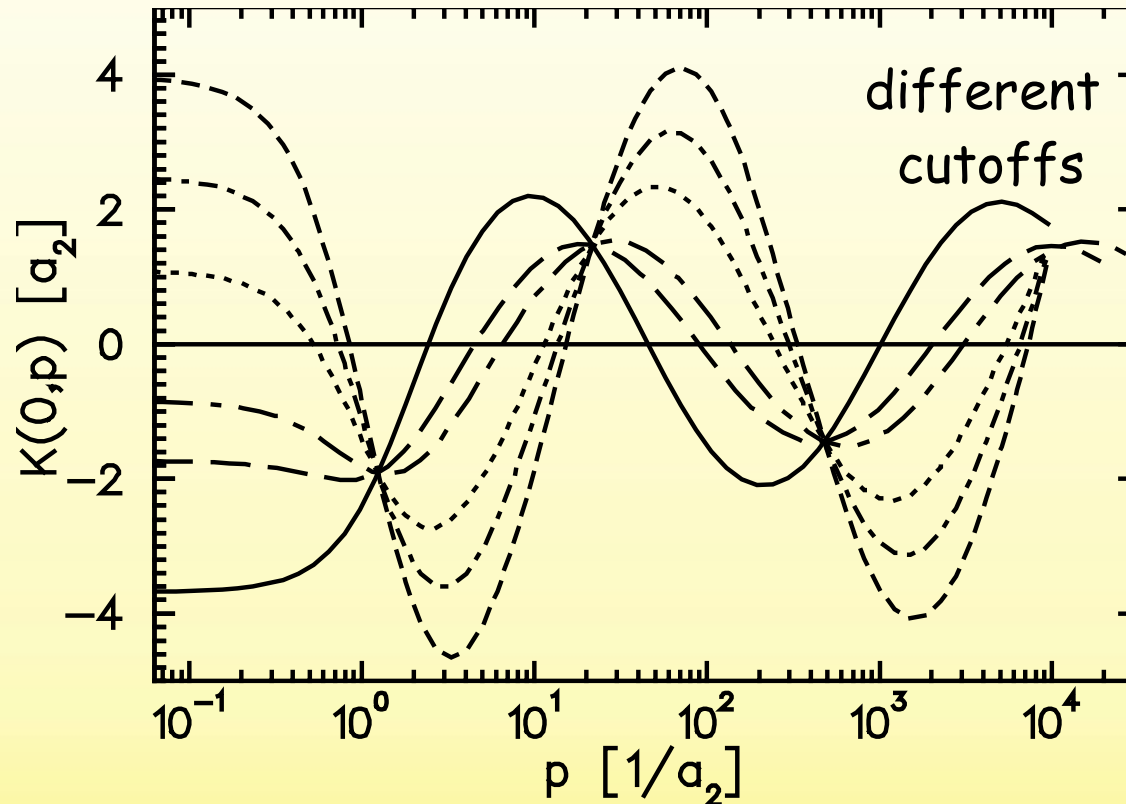
$$a_{Nd,3/2} = 5.09 + 0.89 + 0.35 + \dots \text{ fm} = 6.33 \pm 0.10 \text{ fm}$$

$$a_{Nd,3/2} = 6.35 \pm 0.02 \text{ fm (exp)}$$

QED-like precision!

$$\text{bosons} \quad S = 0 \quad \lambda = 1 \quad \Rightarrow \quad s = \pm i s_0 \quad s_0 \simeq 1.00624$$

Bedaque, Hammer + v.K. '98 '99

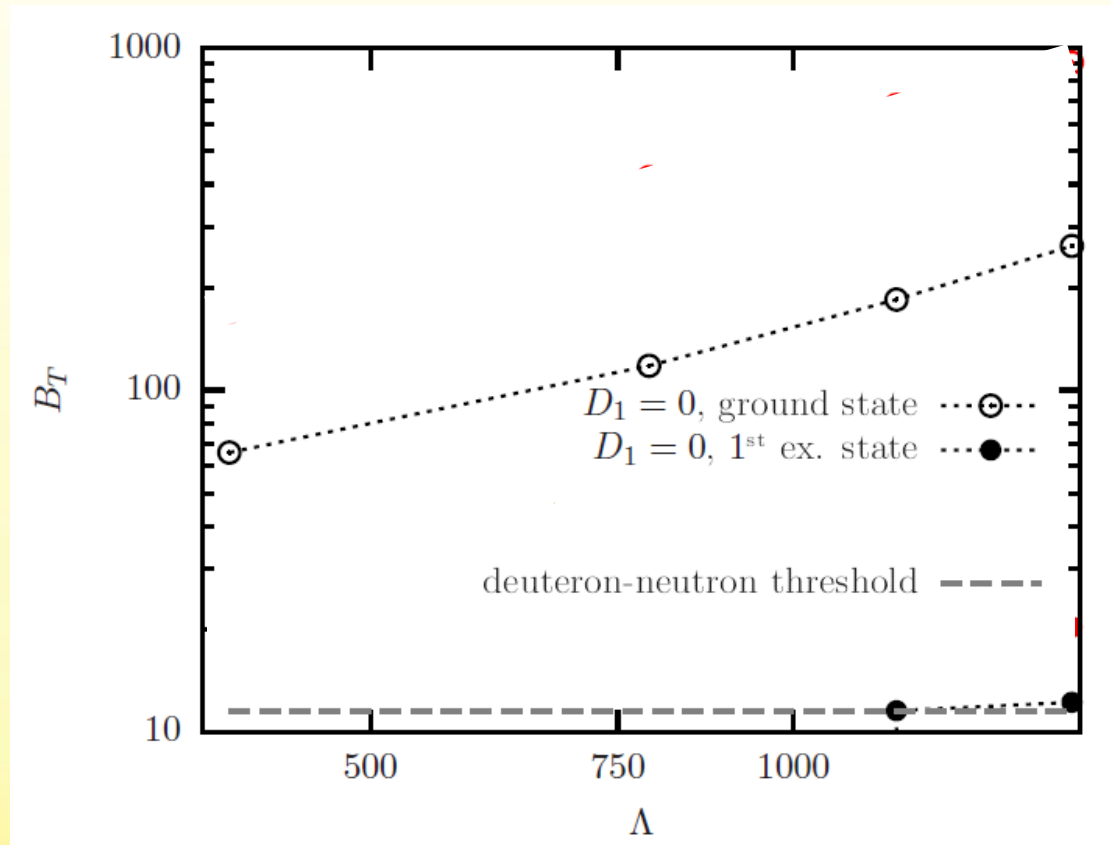


$$\frac{\partial T_{\psi d}(\textcolor{red}{p}'a_2 \sim 1, 0)}{\partial \textcolor{violet}{\Lambda}} \neq 0$$

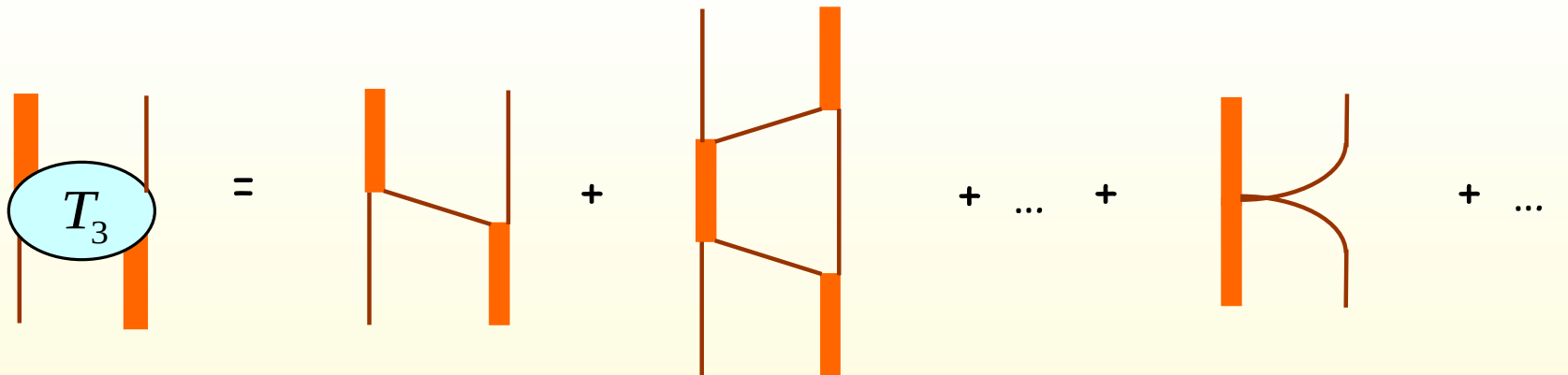
$$T_{\psi d}(\textcolor{blue}{p}'a_2 \gg 1, 0) \propto \cos \left(s_0 \ln \frac{\textcolor{blue}{p}'}{\textcolor{violet}{\Lambda}} + \delta \right)$$

"Thomas collapse"

Thomas '35

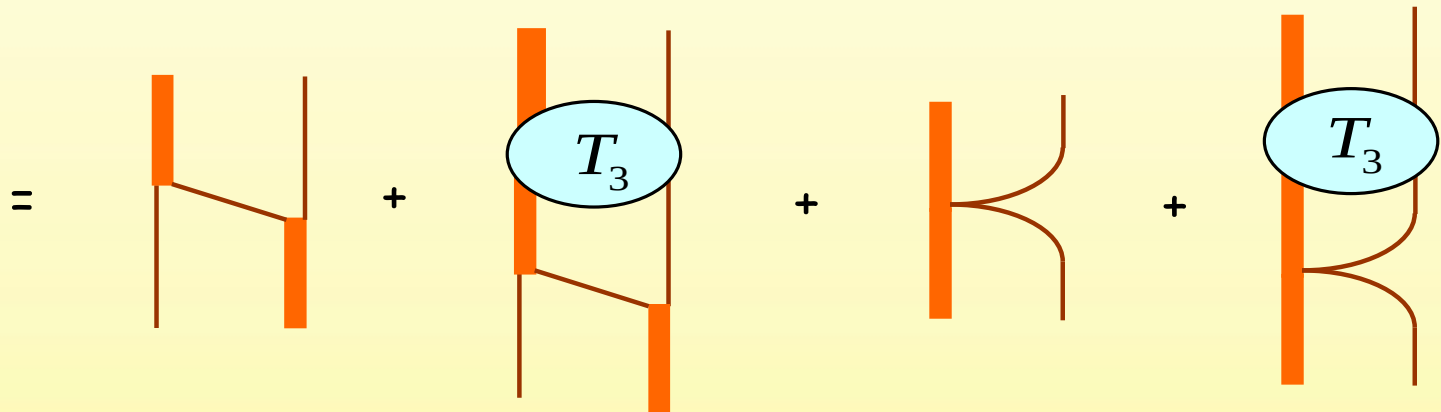


Kirscher, Barnea, Gazit, Pederiva + v.K. '15



$$\sim \frac{4\pi}{Q^2}$$

$$\sim \frac{4\pi}{M_{lo}^2}$$



$$T_3 = K_{OPE} + \lambda \int_0^\Lambda \frac{d^3 l}{(2\pi)^3} \frac{T_3 K_{OPE}}{D} + K_{TBF} + \lambda \int_0^\Lambda \frac{d^3 l}{(2\pi)^3} \frac{T_3 K_{TBF}}{D}$$

Bedaque, Hammer + v.K. '98 '99

Repeat with

$$\begin{cases} \nu_3(\vec{p}', \vec{p}) \rightarrow \nu_3(\vec{p}', \vec{p}) + h_0(\Lambda) \\ \nu_{3,0}(p', k) \rightarrow \nu_{3,0}(p', k) + h_0(\Lambda) \end{cases} \quad h_0(\Lambda) \equiv 8\pi \frac{H(\Lambda)}{\Lambda^2}$$

UV properties: $p' \gg k \gtrsim 1/a_2$

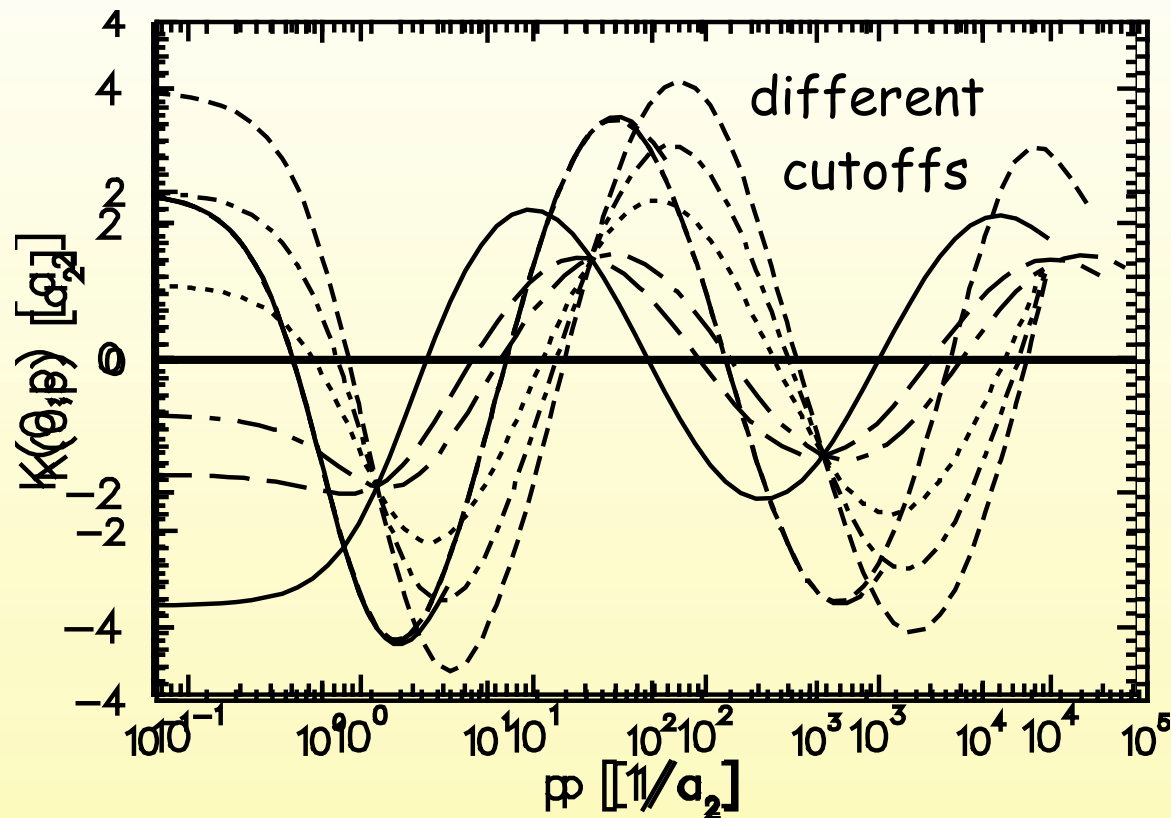
$$T_{3,0}(p' \gg k) = \frac{4}{\sqrt{3}\pi} \int \frac{dl}{p'} \left[\ln \left(\frac{p'^2 + p'l + l^2}{p'^2 - p'l + l^2} \right) - 2 \frac{p'l}{\Lambda^2} H(\Lambda) \right] T_{3,0}(l \gg k)$$

only important for $p' \sim \Lambda$

$$\Rightarrow T_{3,0}(\Lambda \gg p' \gg a_2^{-1}, 0) \propto \cos \left(s_0 \ln \frac{p'}{\Lambda} + \delta(H(\Lambda)) \right)$$

$$\delta(H(\Lambda)) \equiv s_0 \ln(\Lambda/\Lambda_*)$$

physical,
dimensionful
parameter

Fitting to a_3 

$$\frac{\partial T_{\psi d}(p'a_2 \sim 1, 0)}{\partial \Lambda} \neq 0$$

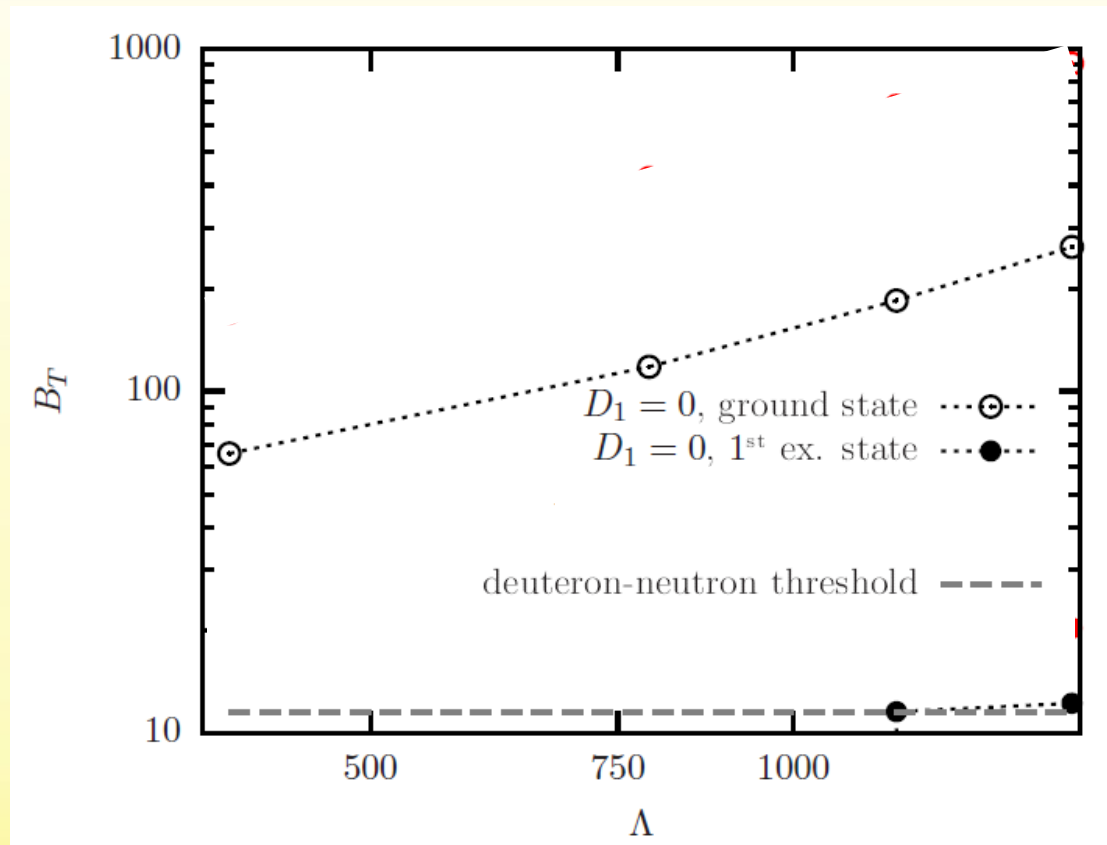
$$T_{\psi d}(p'a_2 \gg 1, 0) \propto \cos\left(\ln\left(\frac{p}{\Lambda}\right) + \delta\right)$$

Renormalized!

"Thomas collapse"

Fitting Λ_* to B_3 :

Thomas '35



outside EFT

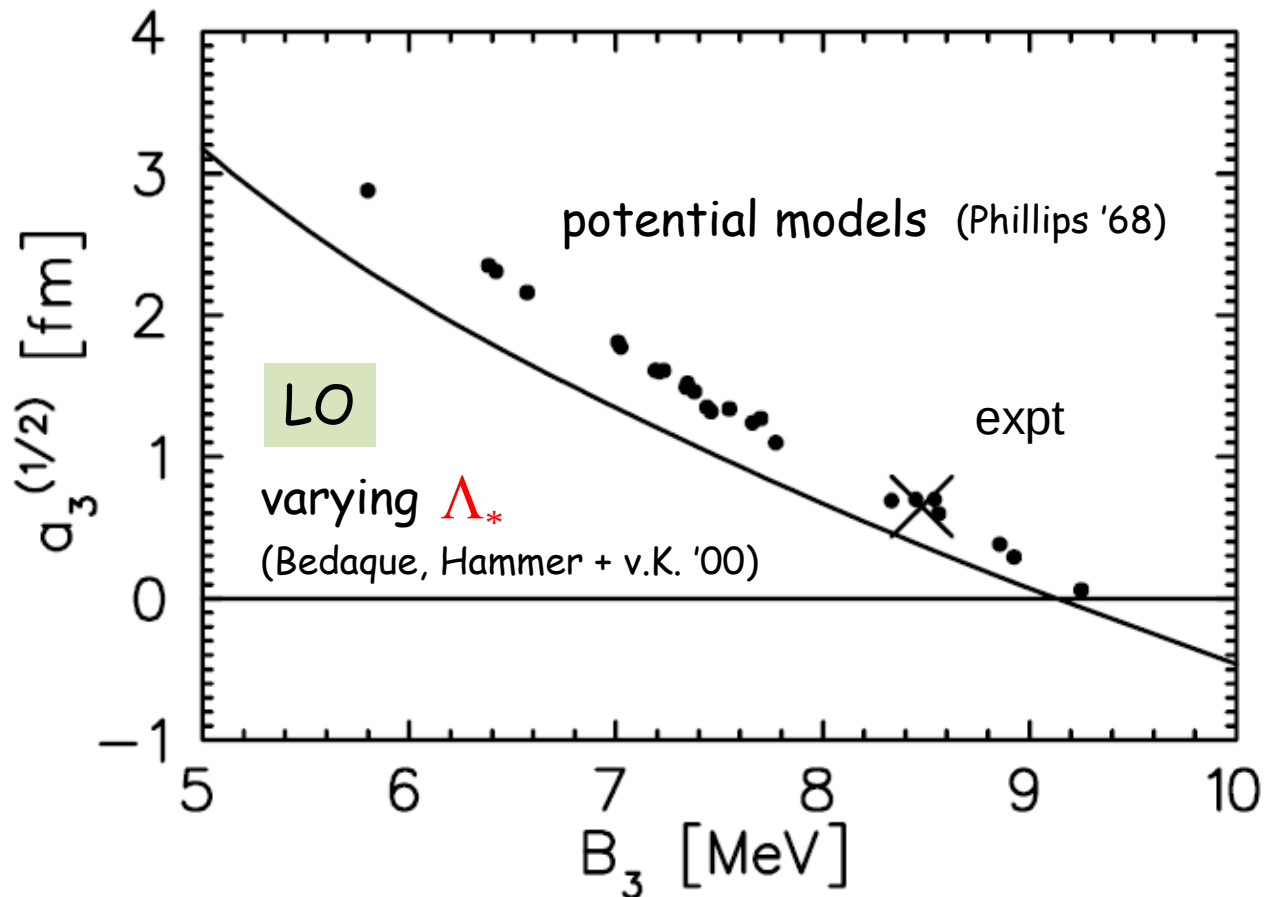
physical
triton

Kirscher, Barnea, Gazit, Pederiva + v.K. '15

Correlations

Phillips line

nucleon-deuteron scattering length



triton binding energy

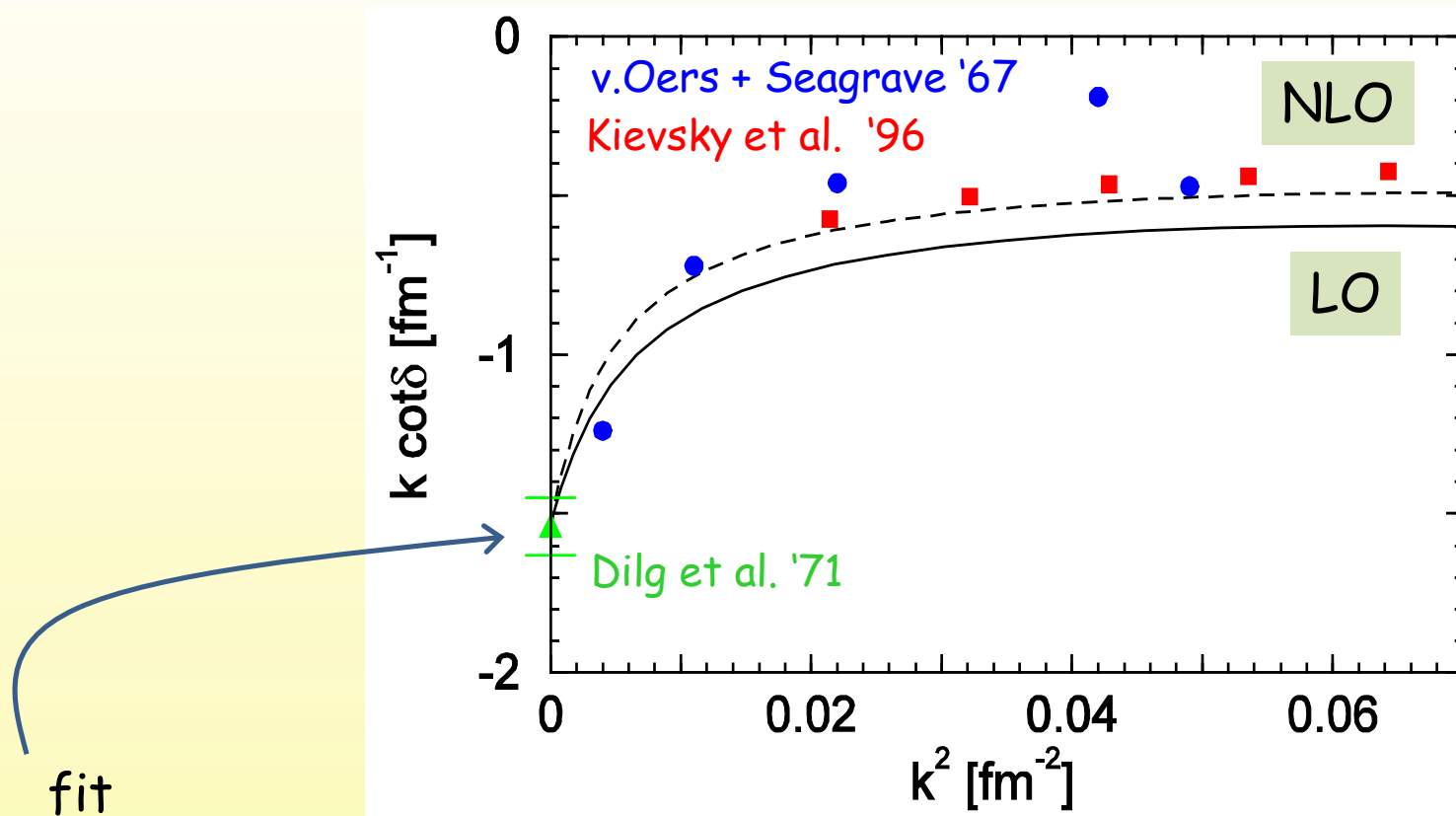
nucleons $S = 1/2$

Bedaque, Hammer + v.K. '99 '00

Hammer + Mehen '01

Bedaque et al. '03

...



$$a_{Nd,1/2} = 0.65 \text{ fm (exp)}$$

postdict

$$B_t = 8.0 + 0.8 + \dots \text{ MeV} = 8.8 \pm 0.5 \text{ MeV}$$

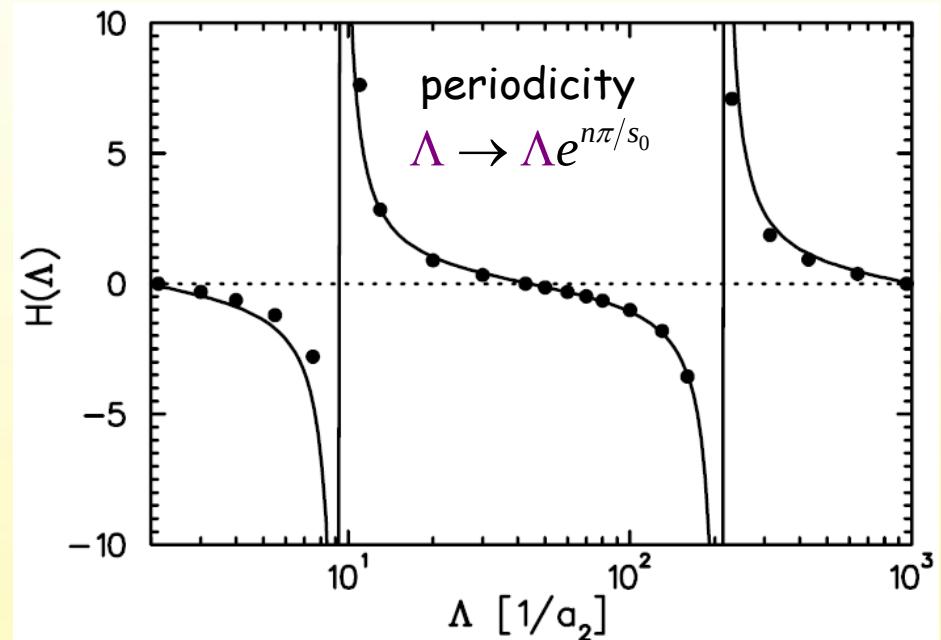
$$B_t = 8.48 \text{ MeV (expt)}$$

Studying UV properties
in more detail:

$$H(\Lambda) \equiv \frac{\Lambda^2 D_0(\Lambda)}{m C_0^2(\Lambda)}$$

$$\simeq \frac{\sin(\ln(\Lambda/\Lambda_*) + \arctan(1/s_0))}{\sin(\ln(\Lambda/\Lambda_*) - \arctan(1/s_0))}$$

Fitting Λ_* to B_3 :



RG limit cycle!

{ bosons
fermions with more than two states

$$D_0 \sim \frac{1}{M_{lo}^4}$$

LO

not just the
effective-range expansion!

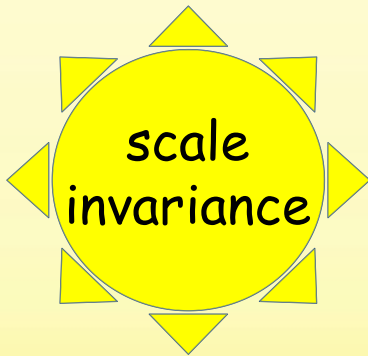
Discrete Scale Invariance

$$A = 2$$

Unitarity limit

$$C_0(\Lambda) = -\frac{1}{\theta_1 \Lambda} \quad \longleftrightarrow \quad m B_2^{(0)} = 0$$

regulator-dependent number



$$x \rightarrow \alpha x$$

$$\Lambda \rightarrow \alpha^{-1} \Lambda$$

$$\frac{t}{m} \rightarrow \alpha^2 \frac{t}{m}$$

$$\psi \rightarrow \alpha^{-3/2} \psi$$

$$S_{EFT}^{(0)} \rightarrow S_{EFT}^{(0)}$$

(includes Wigner spin-isospin symmetry)

$$SU(4)_W$$

Mehen, Stewart + Wise '00
...

$$A = 3$$

$$D_0^{(0)}(\Lambda) \propto \frac{1}{\Lambda^4} \frac{\sin(s_0 \ln(\Lambda/\Lambda_*) + \arctan s_0^{-1})}{\sin(s_0 \ln(\Lambda/\Lambda_*) - \arctan s_0^{-1})}$$

dimensionful parameter
(dimensional transmutation)

discrete
scale
invariance

$$\left. \begin{array}{ll} x \rightarrow \alpha_n x & \Lambda \rightarrow \alpha_n^{-1} \Lambda \\ \frac{t}{m} \rightarrow \alpha_n^2 \frac{t}{m} & \psi \rightarrow \alpha_n^{-3/2} \psi \end{array} \right\} S_{EFT}^{(0)} \rightarrow S_{EFT}^{(0)}$$

$$\alpha_n = \exp(n\pi/s_0) = (22.7)^n \quad n = \dots -1, 0, 1, 2, \dots$$

ground state 

Efimov
states

$$m B_{3n}^{(0)} = k_*^2 \exp(-2n\pi/s_0) \lesssim M_{hi}^2$$

$$\ln\left(\frac{k_*}{\Lambda_*}\right) = \ln \beta, \quad \text{mod } \pi/s_0 \quad \beta \simeq 0.383$$

Braaten + Hammer '06

Efimov '71

...

(includes Wigner spin-isospin symmetry)

$SU(4)_W$

Bedaque, Hammer + v.K. '00

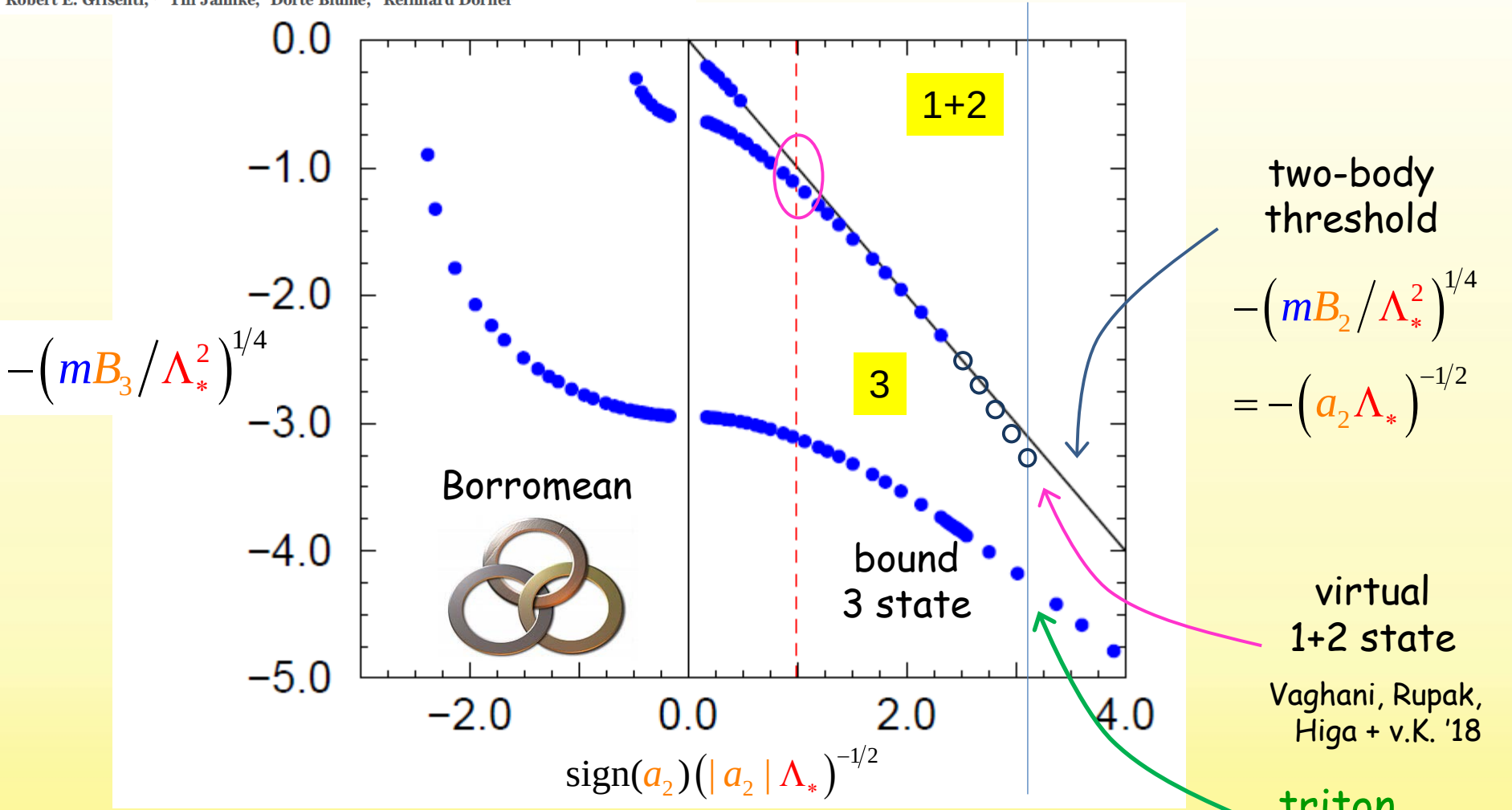
Observation of the Efimov state of the helium trimer

Braaten + Hammer '03

Science 348 (2015) 551

Maksim Kunitski,^{1*} Stefan Zeller,¹ Jörg Voigtsberger,¹ Anton Kalinin,¹
 Lothar Ph. H. Schmidt,¹ Markus Schöffler,¹ Achim Czasch,¹ Wieland Schöllkopf,²
 Robert E. Grisenti,^{1,3} Till Jahnke,¹ Dörte Blume,⁴ Reinhard Dörner^{1*}

⁴He atoms nucleons



decreasing
two-body attraction



INTRODUCTION TO NUCLEAR EFFECTIVE FIELD THEORIES[©]

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*Institut de Physique Nucléaire d'Orsay
and University of Arizona*

Supported in part by
CNRS and US DOE

O u t l i n e

- ❑ Effective Field Theories
- ❑ EFT for Short-Range Forces
- ❑ Two-Body System
- ❑ Three-Body System
- ❑ More-Body Systems
 - ▶ Four bodies
 - ▶ Beyond
- ❑ Long-range Forces

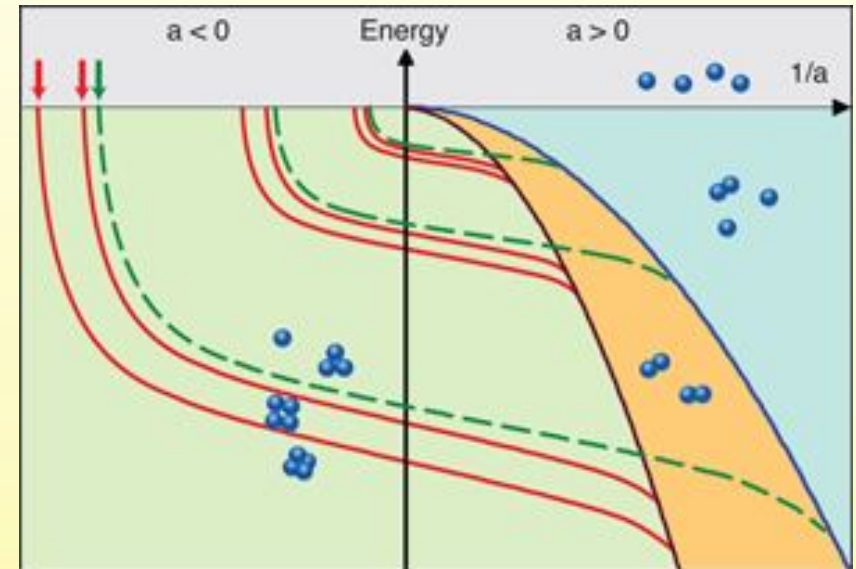
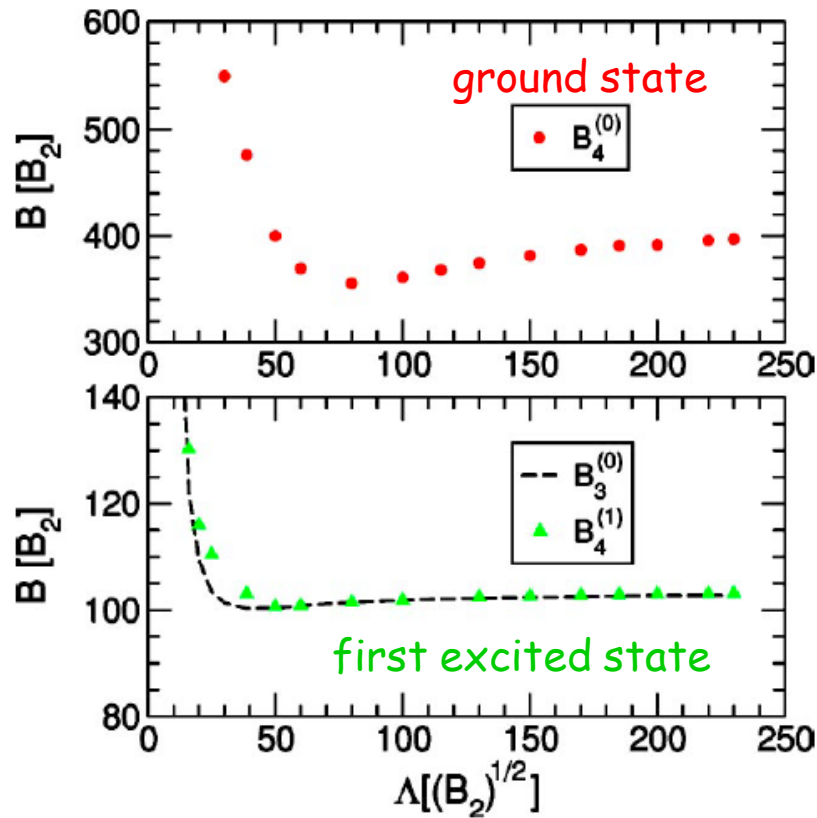
$$A = 4$$

Unitary bosons

Hammer + Platter '07

LO

No 4BF

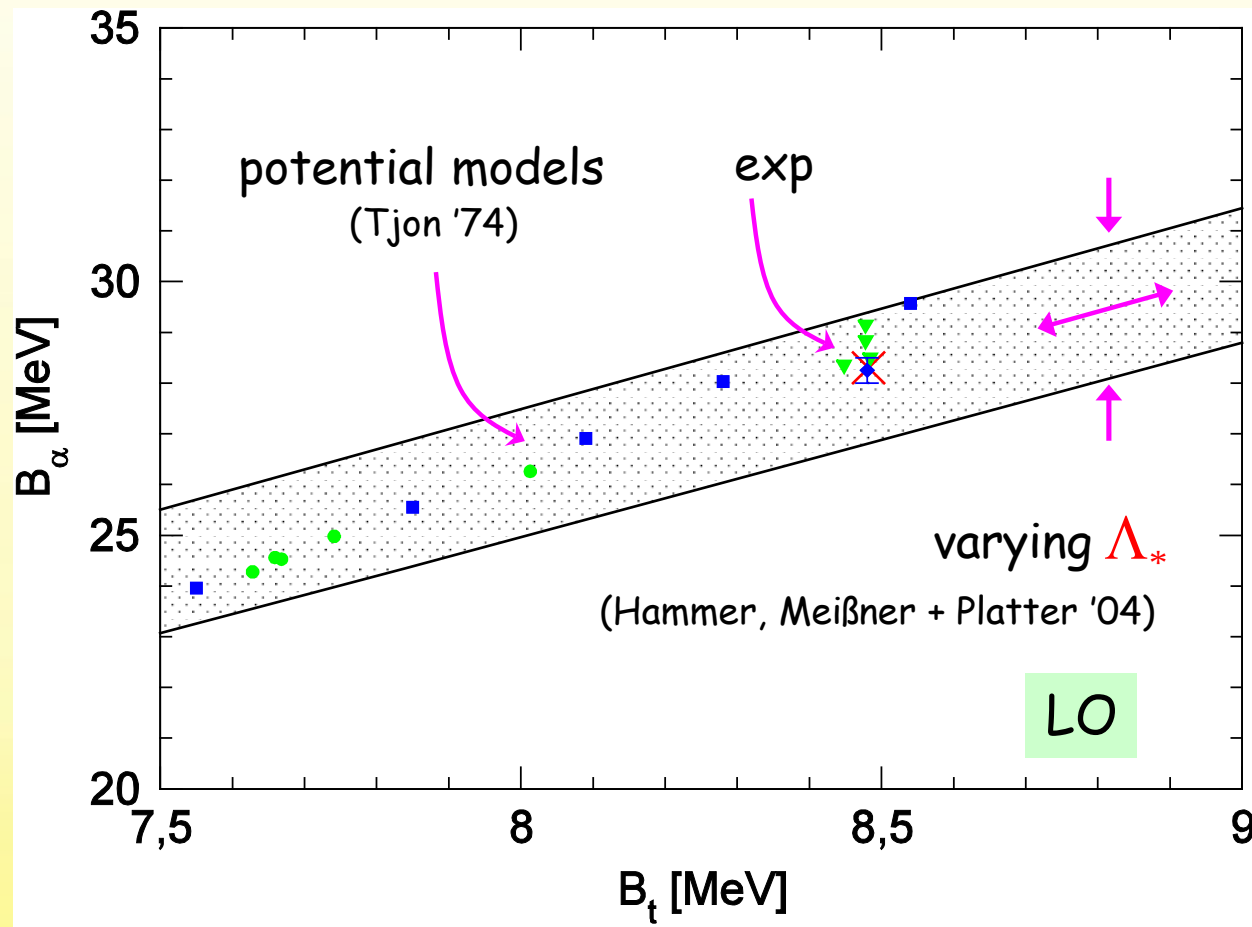


Efimov
descendants

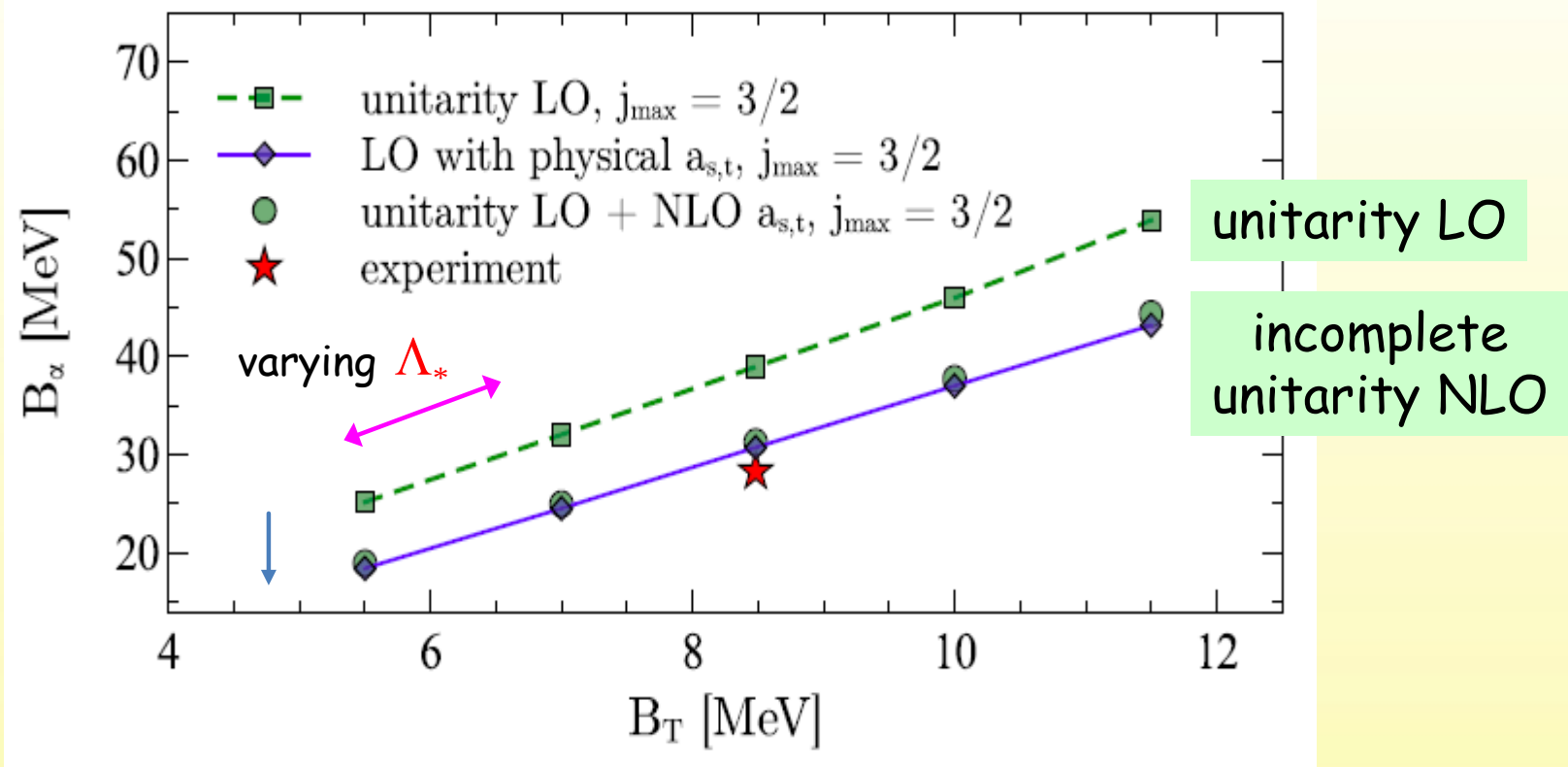
Correlations

nucleons

Tjon line



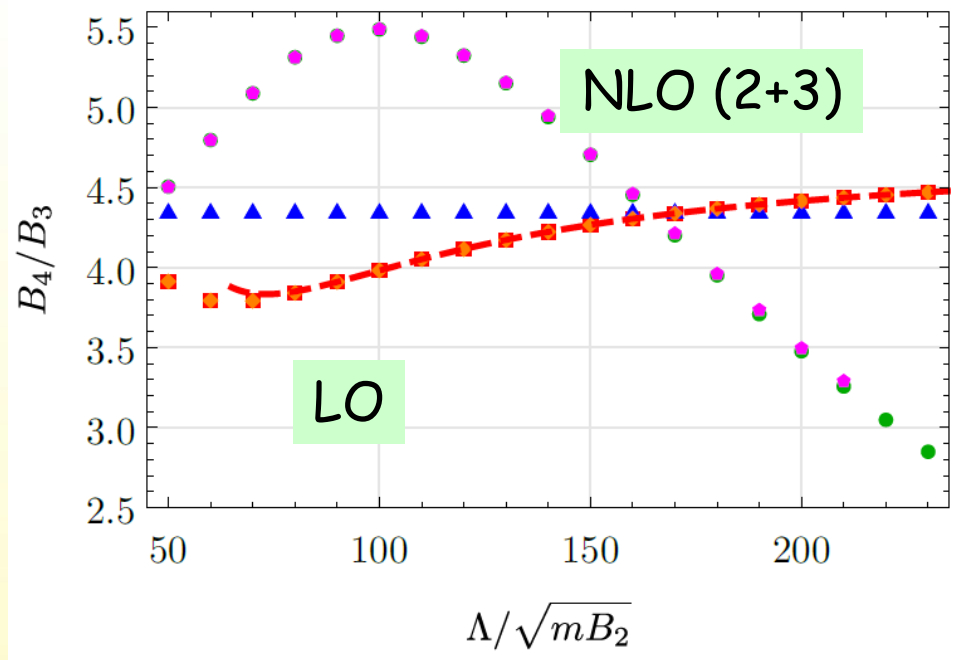
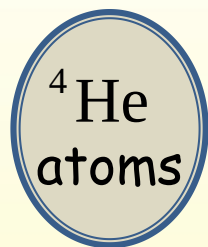
Nucleons at unitarity



Nucleons *perturbatively* close to unitarity



Bosons



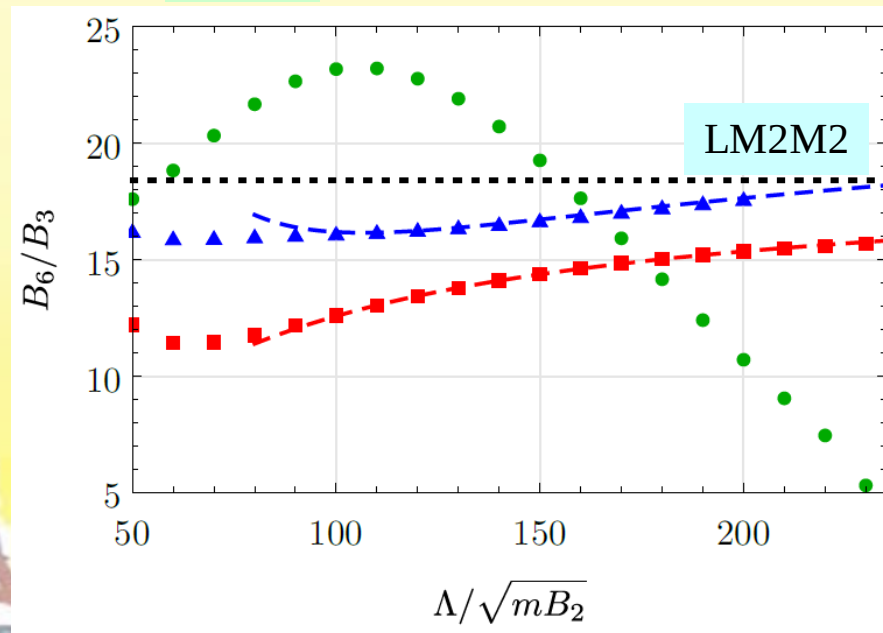
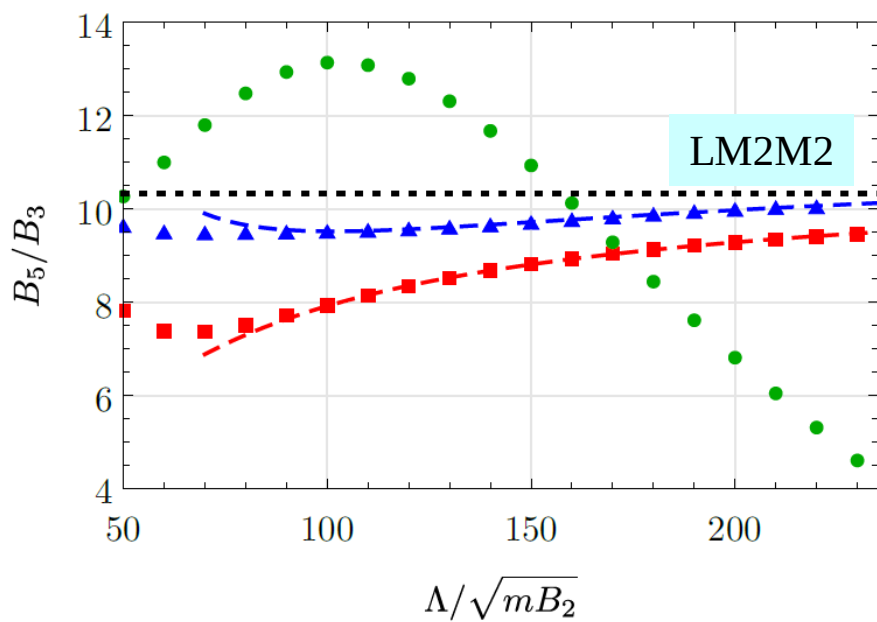
Bazak, Eliyahu + v.K. '16
Bazak, Kirscher, König,
Pavón-Valderrama,
Barnea + v.K. '18

PCKLJS

NLO (2+3+4)

4BF!

No more-body forces up to NLO



Summary:

Expansion parameter $\frac{Q}{M_{hi}}$

- LO: no-derivative two- and three-body interactions

$$C_0, D_0$$

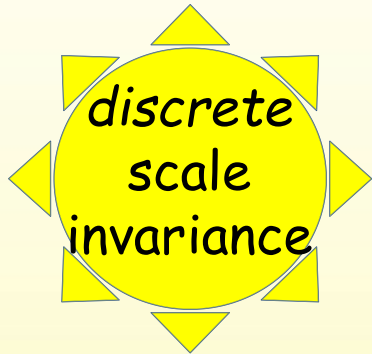
Around unitarity: a SINGLE physical parameter Λ_*

- NLO: two-derivative two- and no-derivative four-body interactions

$$C_2, E_0$$

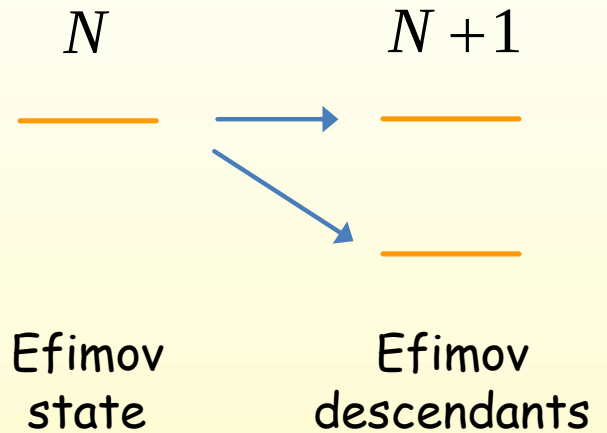
- *Etc.*

Unitary bosons



doubling

Hammer + Platter '06
von Stecher '10'11
Gattobigio, Kievsky + Viviani '11'12



Ground states

single scale

LO



$$\frac{B_N^{(0)}(\Lambda_*)}{N} = \kappa_N \frac{B_3(\Lambda_*)}{3}$$

universal numbers

$$\kappa_2 \equiv 0$$

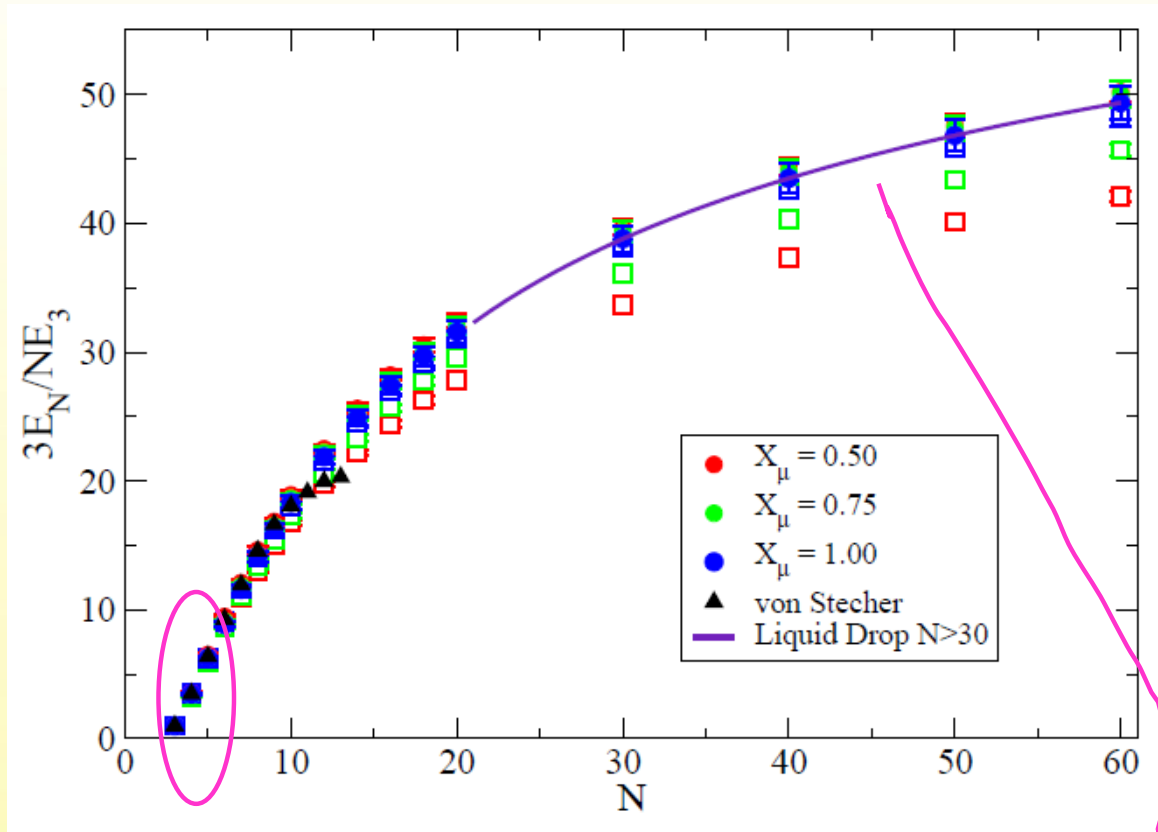
$$\kappa_3 \equiv 1$$

$$\kappa_4 \approx 3.5$$

$$\kappa_{N \geq 5} \approx ?$$



LO



$$\Lambda_2/Q_3 \approx 26 - 38$$

$$\Lambda_3/\Lambda_2 = 0.5 - 1$$

saturation!

cf. Piatecki + Krauth '14
Comparin + Krauth '16

$$\kappa_N \approx \frac{3}{N} (N-2)^2$$

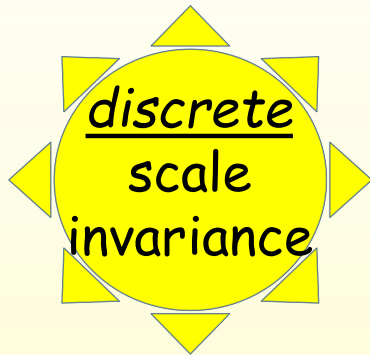
Bazak, Eliyahu + v.K. '16

$$\kappa_N = \kappa_\infty \left[1 + \eta N^{-1/3} + \mathcal{O}(N^{-2/3}) \right]$$

$$\kappa_\infty = 90 \pm 10 \quad \eta = -1.7 \pm 0.3$$

$$A \geq 4$$

Nucleons around unitarity



doubling?

Ground states

single scale

LO



$$\frac{B_A^{(0)}(\Lambda_*)}{A} = \kappa_A \frac{B_3(\Lambda_*)}{3}$$

$$\kappa_2 \equiv 0$$

$$\kappa_3 \equiv 1$$

$$\kappa_4 \approx 3.5$$

$$\kappa_{A \geq 5} \approx ?$$

same as for bosons
should grow slower
than bosons

same
saturation mechanism
as for bosons?



Larger nuclei?

➡ Adapt *ab initio* many-body methods

- ❖ ^4He excited state + ^6Li at LO with no-core shell model
Stetcu, Barrett + v.K. '07
- ❖ ^{16}O at LO with auxiliary-field diffusion Monte Carlo
Contessi et al '17
- ❖ ^4He at LO + resummed NLO with resonating group
and stochastic variational
Kirscher et al. '09; Lensky, Birse + Wallet '15
- ❖ ^4He , ^{16}O , ^{40}Ca at LO + resummed NLO
❖ Bansal et al. '17

+ many applications to exotic hadronic states, cold atoms, ...

$$B_3 = 8.48 \text{ MeV}$$

$$B_3 = 2.28 \cdot 10^{-5} \text{ eV}$$

$3B_A/(AB_3)$	nucleons			${}^4\text{He}$ atoms		
A	experiment	LO EFT	unitarity	potential	LO EFT	unitarity
2	0.39	0.39	0	0.0156	0.0152	0
3	1	1	1	1	1	1
3*	(0.17)	(0.19)	0.0019	0.0180	0.0175	0.0019
4	2.50	2.6	3.5	3.3	3.2	3.46
4*	0.71	0.90	0.75	0.755	0.759	0.752
5	1.94	?	?	6.2	5.7	6.3
6	1.89	1.4	?	9.2	8.2	8.9
$\vdots$						
16	2.82	2.5	?	?	?	27.4
$\vdots$						
$\rightarrow \infty$	5.7	?	?	180	?	90

$= \kappa \left(1 + \eta A^{-1/3} + \dots \right)$ "liquid-drop formula" v. Weizsäcker '36

INTRODUCTION TO NUCLEAR EFFECTIVE FIELD THEORIES[©]

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O u t l i n e

- ❑ Effective Field Theories
- ❑ EFT for Short-Range Forces
- ❑ Two-Body System
- ❑ Three-Body System
- ❑ More-Body Systems
- ❑ Long-range Forces

Next time...



Conclusion

- ◆ Systems near unitarity can be described model-independently by Pionless EFT

[properties given by essentially **one** parameter Λ_*
details obtained in perturbation theory

cf. parallel results from a model perspective: Kievsky *et al.*

- ◆ How far can we go for nuclei?

[more nucleons	Gezerlis <i>et al.</i> , in progress
nuclear matter	Lyu <i>et al.</i> , in progress
higher orders	Bazak <i>et al.</i> , in progress