



Unveiling the Secrets of Nature's Primordial Liquid

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The ubiquitousness of fluid dynamics

Cosmological scale



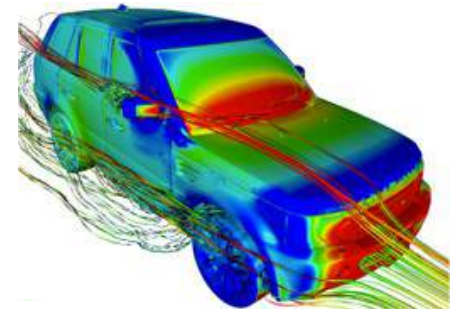
Neutron star mergers



Iguaçu Falls

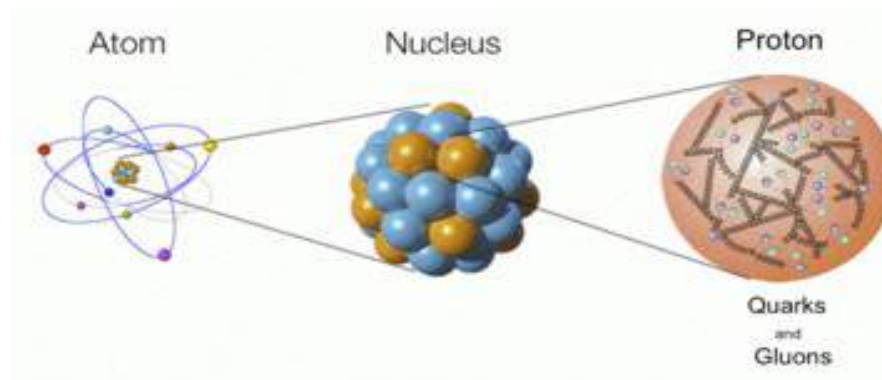


Human scale



Quantum Mechanics + Special Relativity

This talk:



Liquid?

The ubiquitousness of fluid dynamics

Based on conservations laws + large separation of length scales

Separation of scales → macroscopic: L microscopic: ℓ

Knudsen number

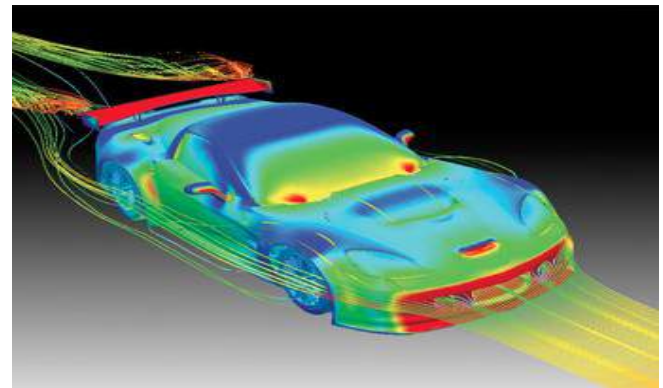
$$K_N \sim \frac{\ell}{L} \ll 1$$



FLUID



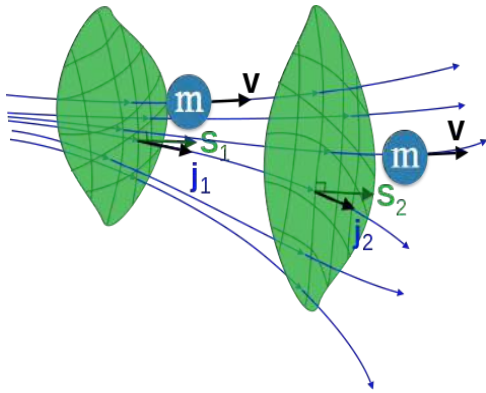
$$L \sim 10^5 \text{ m} \quad \ell \sim 1 \text{ m}$$



$$L \sim 1 \text{ m} \quad \ell \sim 10^{-7} \text{ m}$$

How does one describe fluid dynamics?

Conservation of mass + Newton's 2nd law + isotropy + incompressibility (ρ_0)



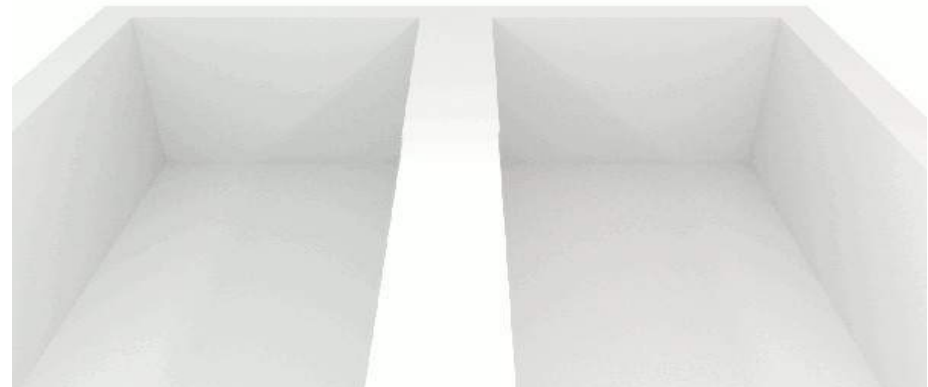
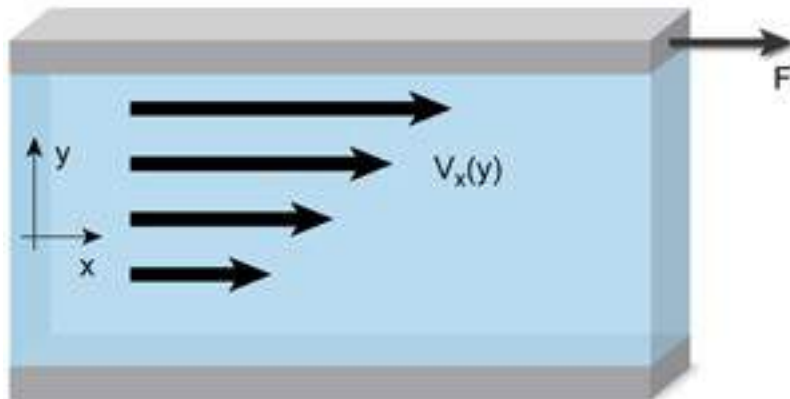
$$\underbrace{\partial_t \vec{V} + (\vec{V} \cdot \vec{\nabla}) \vec{V}}_{\text{Ideal fluid}} + \underbrace{\frac{\vec{\nabla} P}{\rho_0}}_{\sim \mathcal{O}(K_n)} = \underbrace{\frac{\eta}{\rho_0} \nabla^2 \vec{V}}_{\sim \mathcal{O}(K_n)} + \underbrace{\mathcal{O}(K_N^2)}_{\text{Higher order}}$$

Ideal fluid $\sim \mathcal{O}(K_n^0)$

$\sim \mathcal{O}(K_n)$

Higher order

$\eta \rightarrow$ shear viscosity



ideal fluid

viscous fluid



Navier-Stokes Equations

Valid when $K_N \ll 1$

Navier

$$\partial_t \vec{V} + (\vec{V} \cdot \nabla) \vec{V} + \frac{\nabla P}{\rho_0} = \frac{\eta}{\rho_0} \nabla^2 \vec{V}$$



Stokes
~ 1845

- Notoriously hard nonlinear problem to solve in three dimensions.

- Turbulence



da Vinci, 1508-1513

Millennium Prize Problem



“Existence and smoothness of solutions of the Navier-Stokes equations”

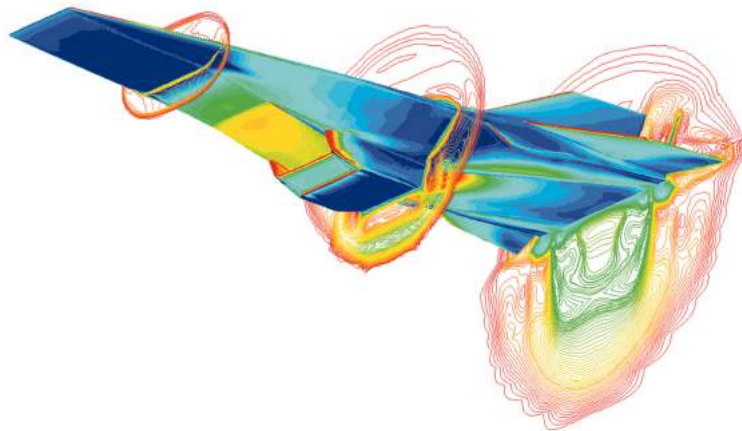


What are the limits of fluid dynamic behavior?

Macroscopic: gradient of velocity field $\rightarrow \partial V \sim 1/L$

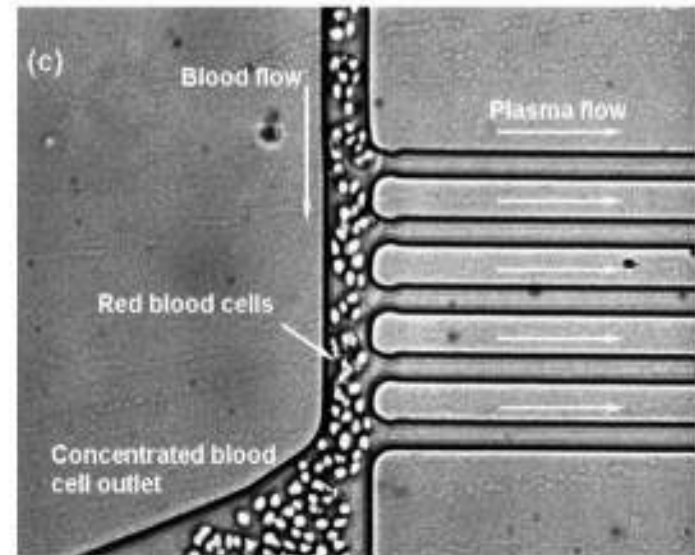
Microscopic scale (mean free path) $\rightarrow \ell_{mfp} \sim 1/(n\sigma)$

Rarefied gases $\rightarrow K_N \sim \frac{\ell}{L} \sim 1$



High-altitude flights

$$L \sim 10^{-6} \text{ m}$$

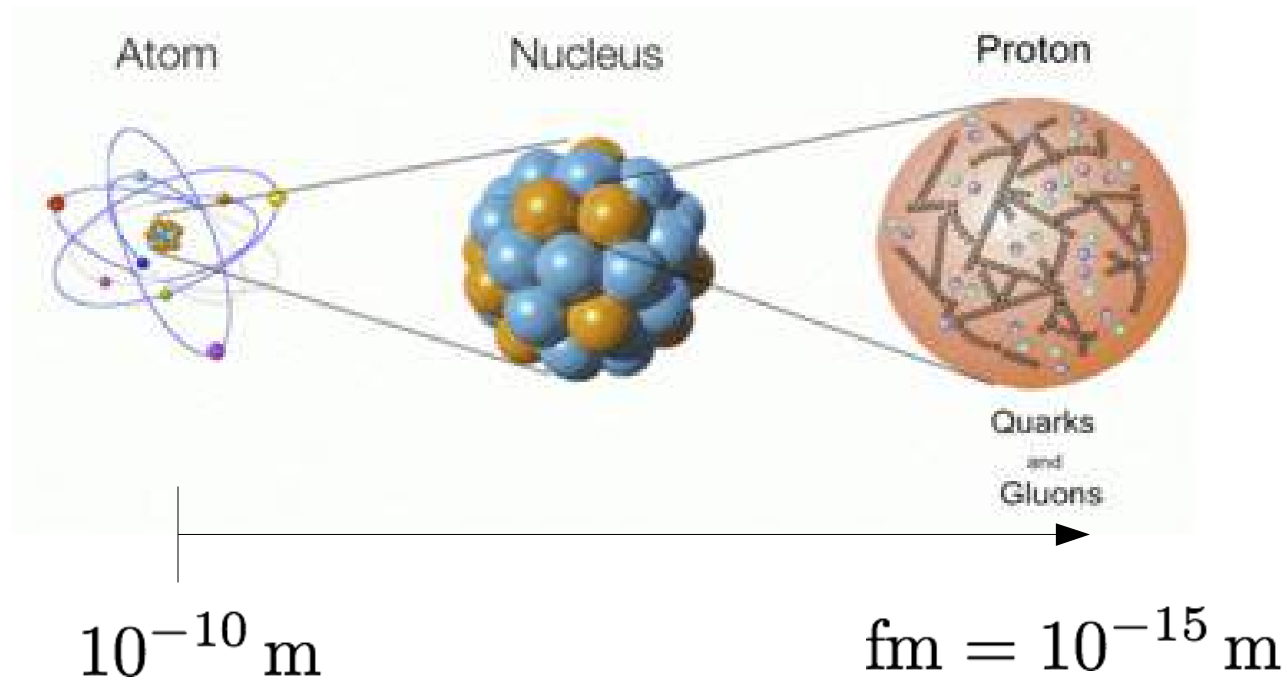


Yang et al, Lab on Chip, 2006

Microfluidic devices in medicine

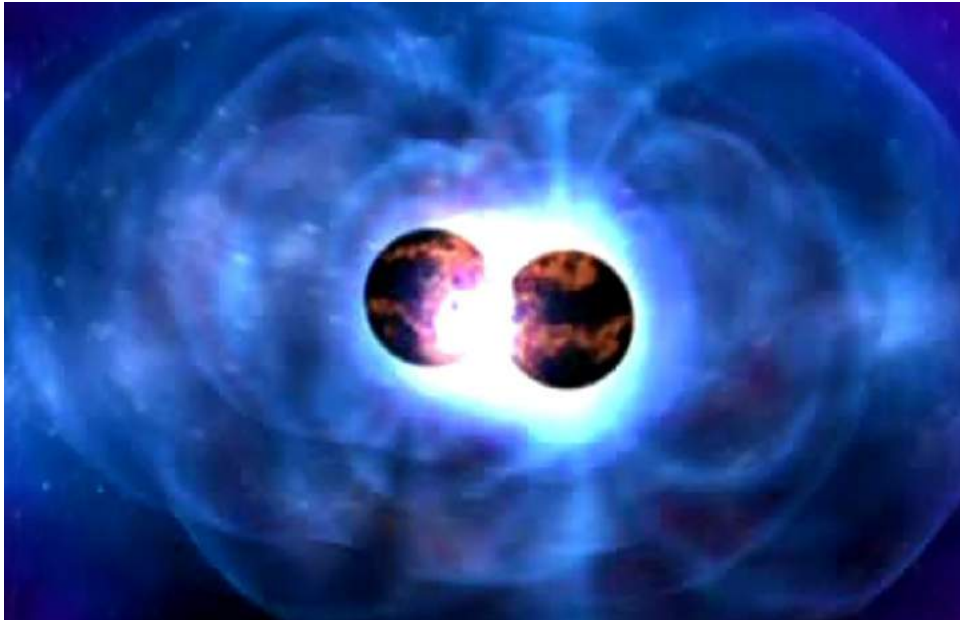
The Frontiers of Fluid Dynamics

Redefine “Macro” Scales → Nuclear/Particle Physics



The Frontiers of Fluid Dynamics

Viscous Fluid Dynamics in Strong Fields



Neutron Star Mergers

Quantum Chromodynamics (QCD)

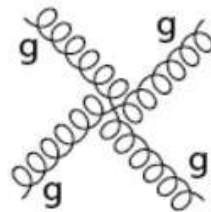
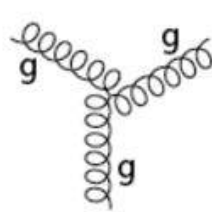
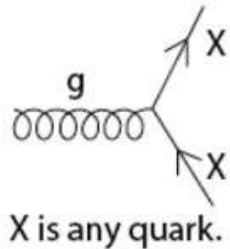


2004

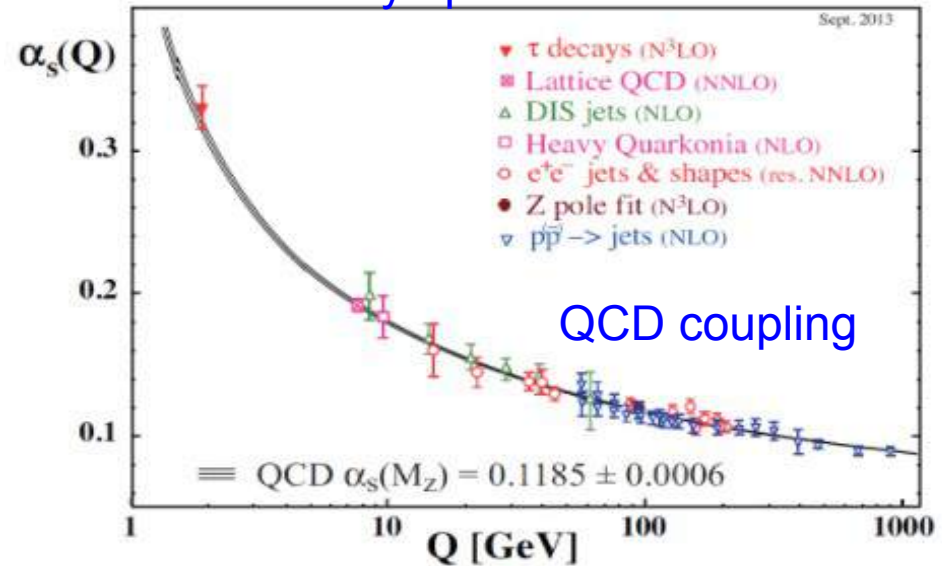
The fundamental theory of the strong interactions

Asymptotic freedom

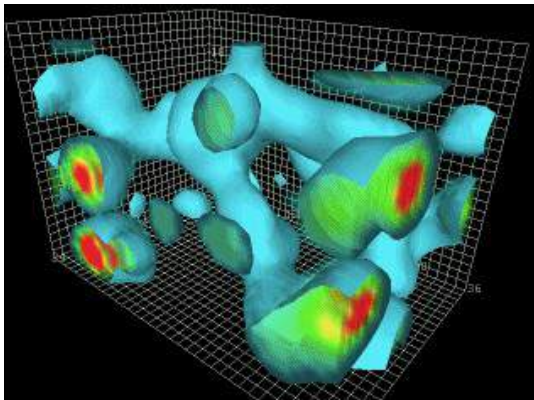
Non-Abelian gauge theory



gluon self-interactions



The QCD vacuum

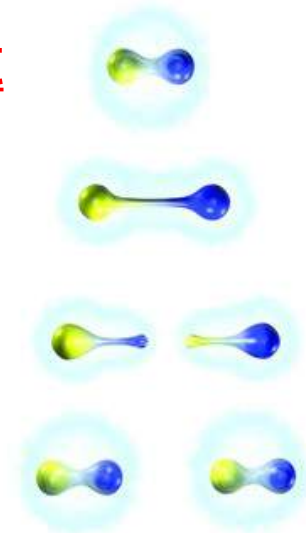


from D. B. Leinweber

Color confinement

Strong coupling
phenomenon !!!

Quarks and gluons
can never be truly free

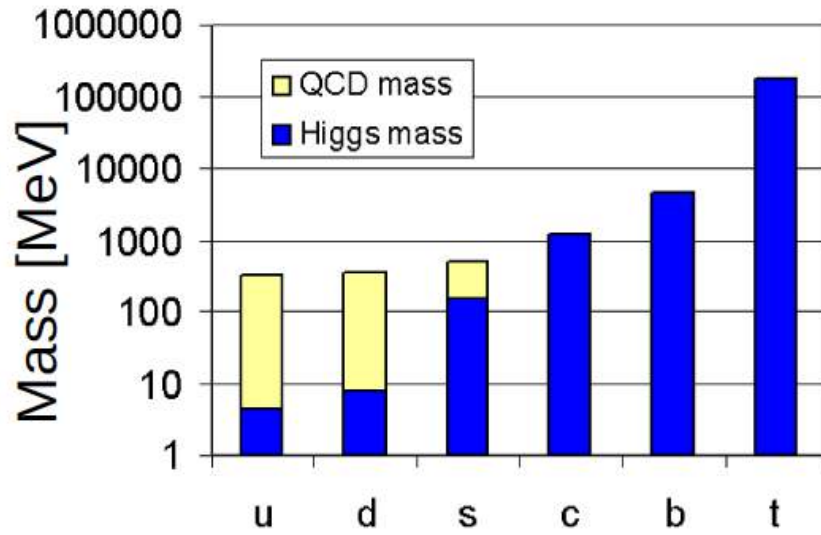


Hadron
(pion)

Quantum Chromodynamics (QCD)

The fundamental theory of the strong interactions

QCD is $\sim 95\%$ of visible (baryonic) matter



B. Mueller Nucl.Phys. A750 (2005) 84-97



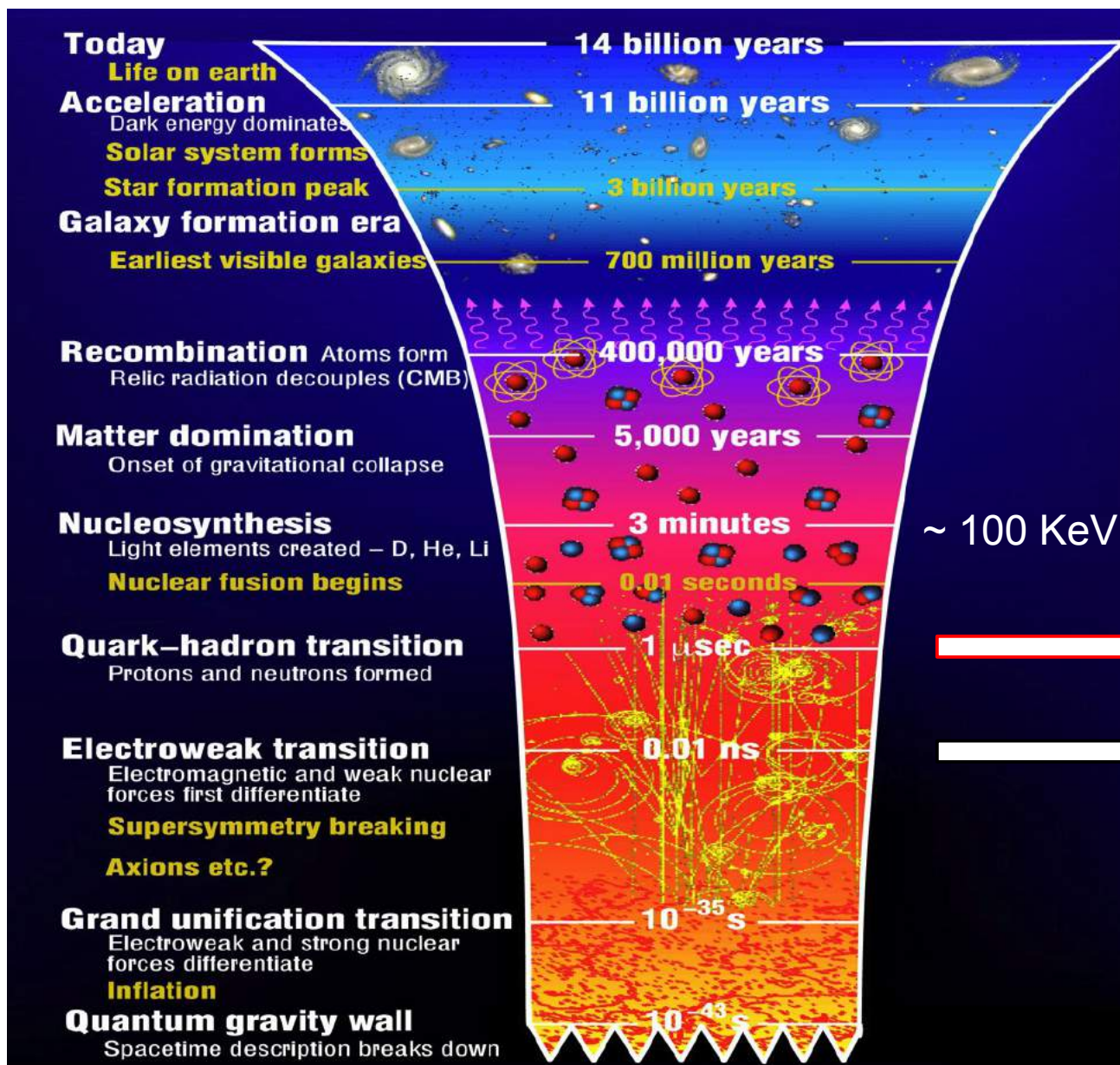
At temperatures $T \sim m_\pi \sim 150 \text{ MeV} \sim 10^{12} \text{ K}$

Deconfinement: Quarks and gluons not confined into hadrons

Collins and Perry, PRL 1975

→ **Early universe!**

Deconfined QCD matter → Quark-Gluon Plasma (QGP)



~ 100 KeV

QCD

$$T_c \sim 155 \text{ MeV}$$

$$T_c \sim 159 \text{ GeV}$$

$$m_H = 125 \text{ GeV}$$

Hot QCD from first principles – Lattice formulation

Statistical mechanics of quarks and gluons

QCD partition function

$$Z = \int \mathcal{D}A_\mu^a \mathcal{D}\bar{\psi} \mathcal{D}\psi \exp\{-S(A, \bar{\psi}, \psi)\}$$

Continuum formulation

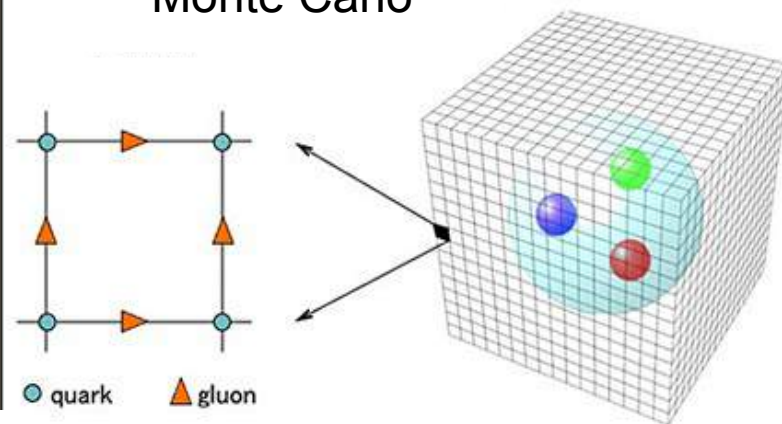
$$\mathcal{L} = \frac{1}{4g^2} G_{\mu\nu}^a G_{\mu\nu}^a + \sum_j \bar{q}_j (i\gamma^\mu D_\mu + m_j) q_j$$

where $G_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + if_{bc}^a A_\mu^b A_\nu^c$
and $D_\mu \equiv \partial_\mu + it^a A_\mu^a$

That's it!

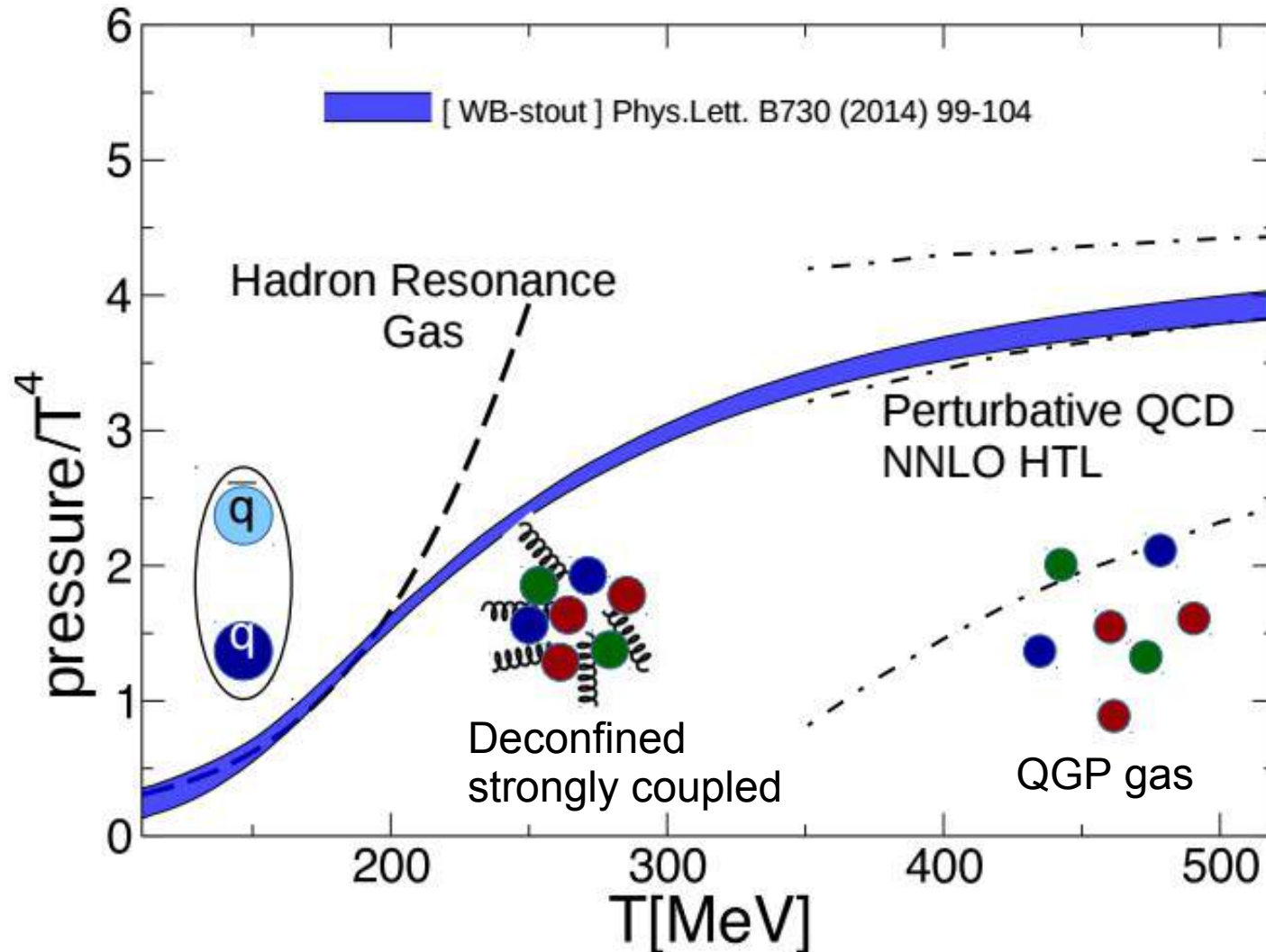
Lattice formulation

Monte Carlo



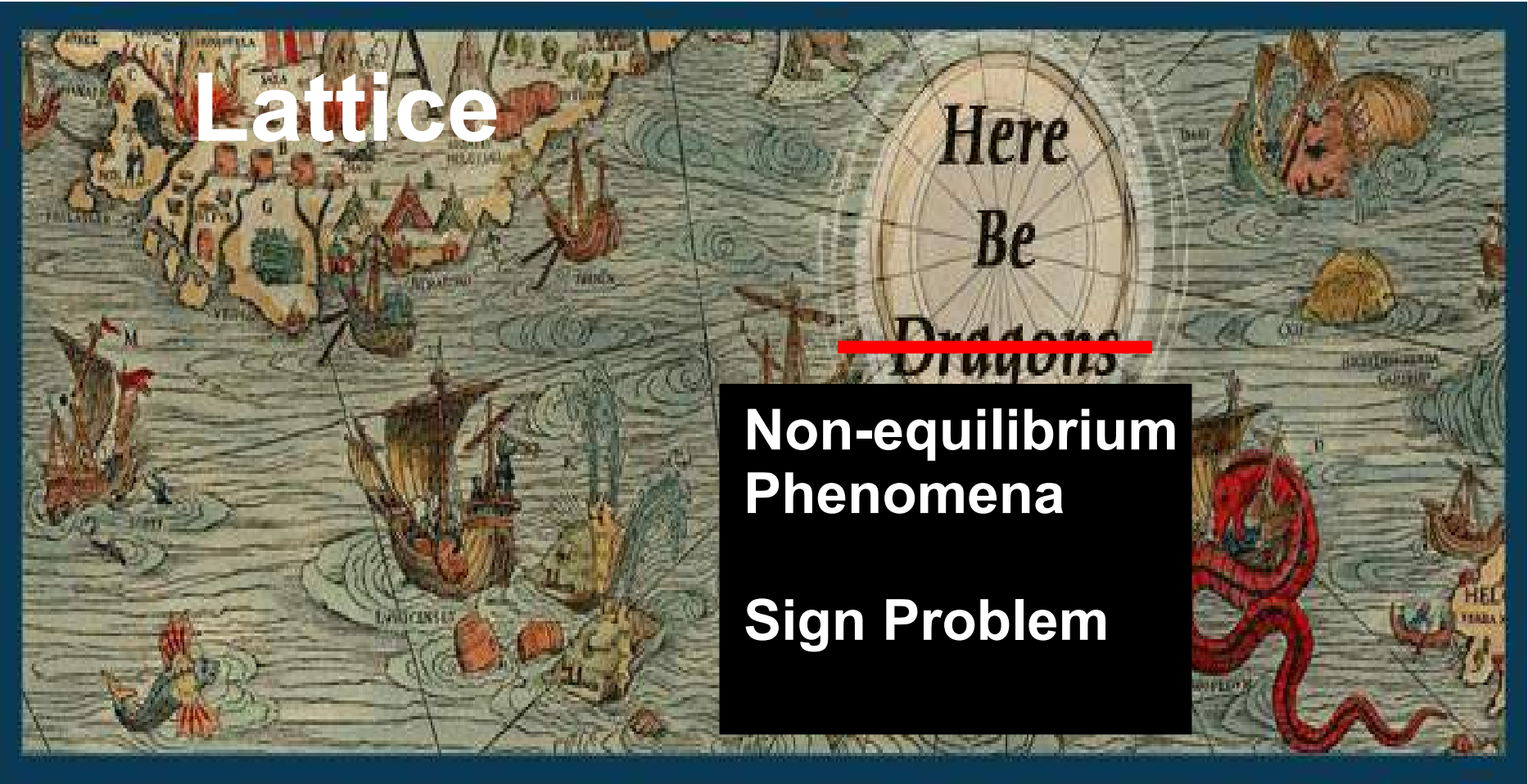
QGP in thermodynamic equilibrium

QCD phase transition in the early universe was a crossover



Non-equilibrium properties of non-Abelian gauge fields?

Relevant for nuclear physics, astrophysics, and cosmology

A historical map, likely a 15th-century portolan chart, featuring a compass rose in the center. The map is filled with various sea monsters and mythical creatures, including a large red dragon on the right, a sea monster on the left, and several other creatures in the water. The text 'Lattice' is overlaid on the left side of the map.

Lattice

**Here
Be
Dragons**

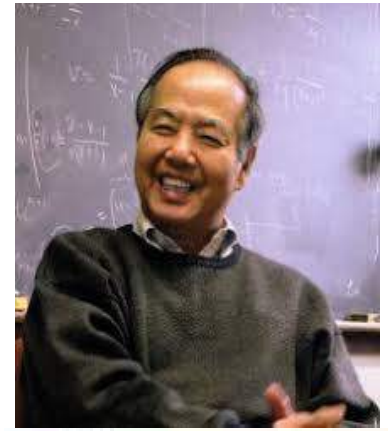
**Non-equilibrium
Phenomena**

Sign Problem

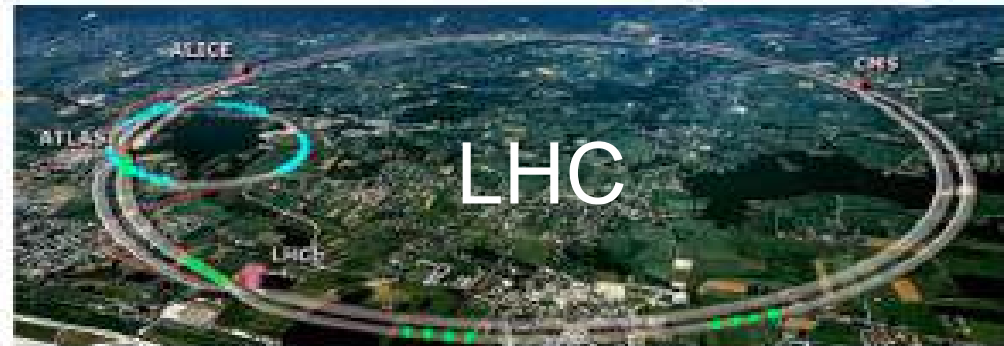
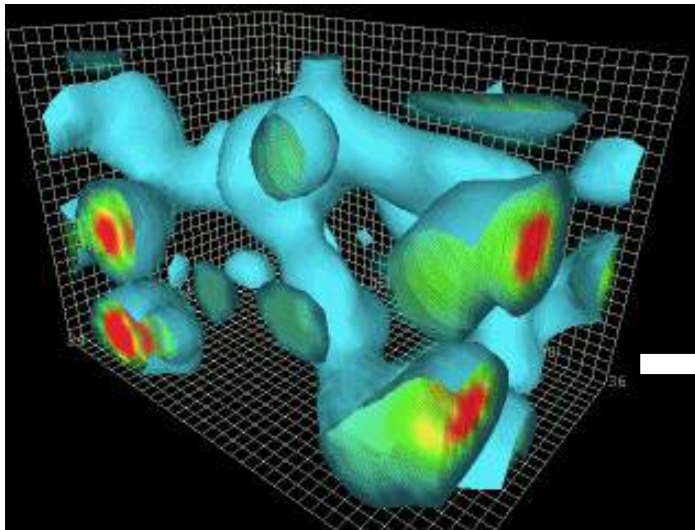
Out-of-equilibrium properties of QCD?

*In order to study the question of the QCD “vacuum”, we must turn to a different direction, we should investigate some “**bulk**” phenomena by distributing **large energy over a large volume**.*

T. D. Lee, Rev. Mod. Phys. 47 (1975)

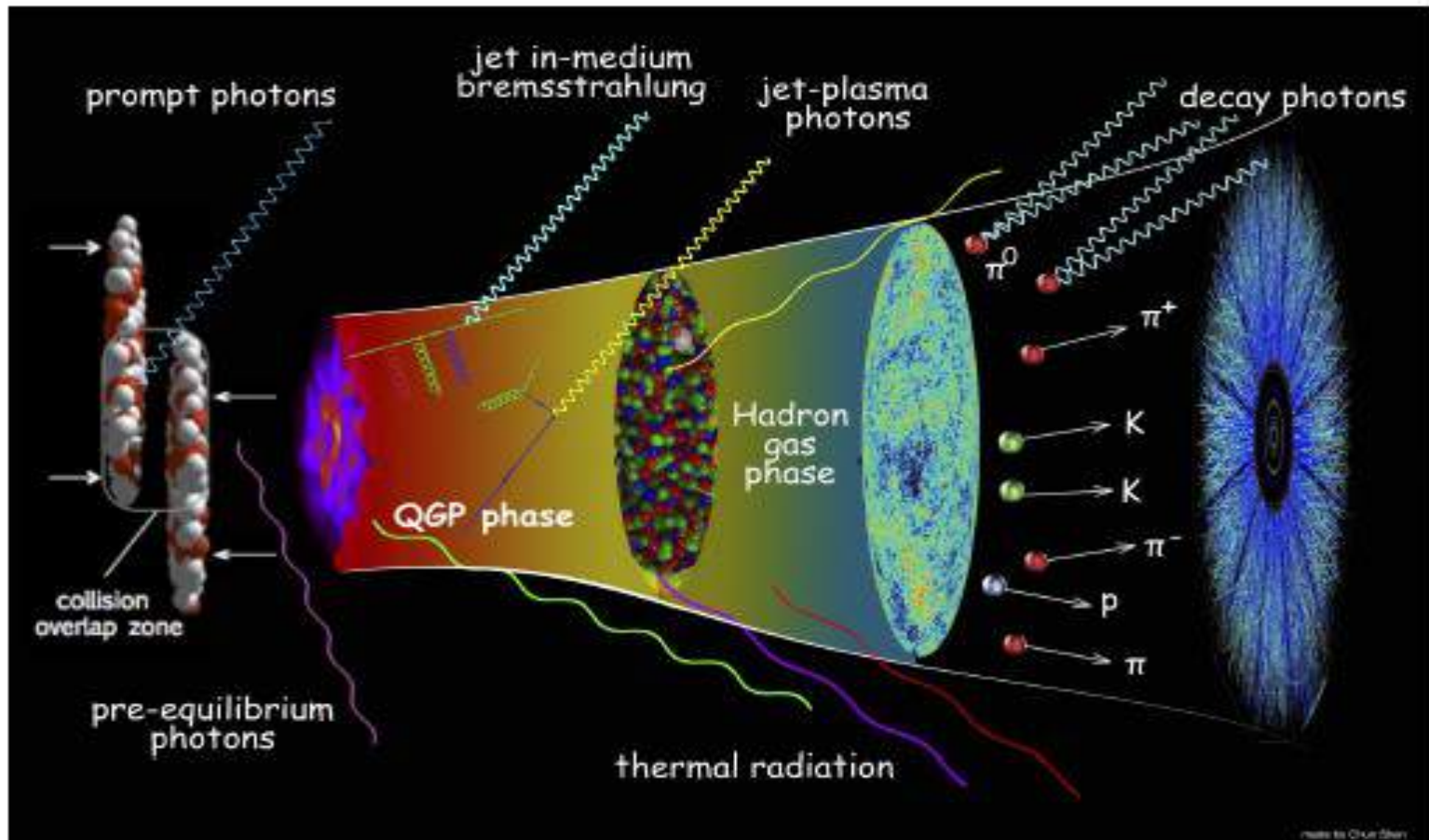


QCD vacuum $T = 0$



Heavy Ion Collisions in a Nutshell – The Little Bang

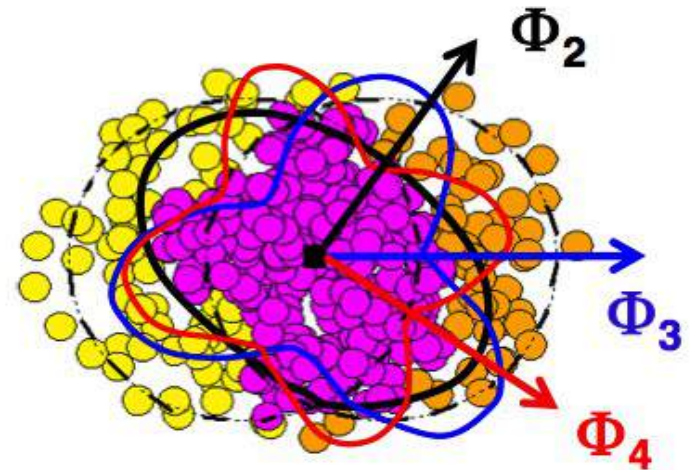
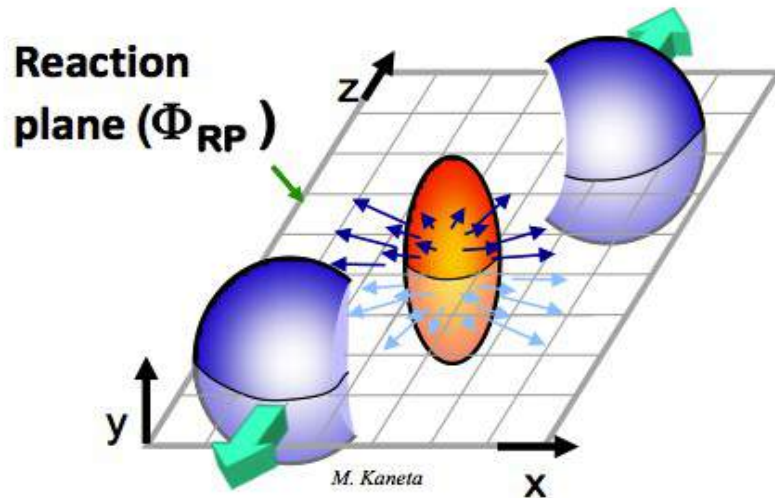
The way to study non-equilibrium hot QCD phenomena in the lab



QGP = The hottest, densest, smallest, most perfect liquid

Main observable for bulk properties: Flow anisotropies

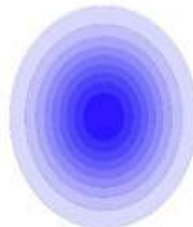
Strongly interacting QGP



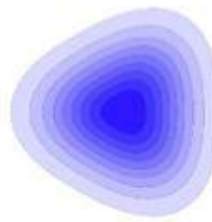
Particles produced

$$E \frac{d^3N}{d^3p} = \frac{1}{2\pi} \frac{d^2N}{p_T dp_T dy} \left[1 + \sum_n 2v_n \cos[n(\phi - \psi_n)] \right]$$

Flow harmonics



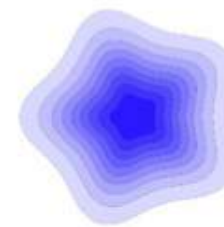
$n = 2$



$n = 3$



$n = 4$



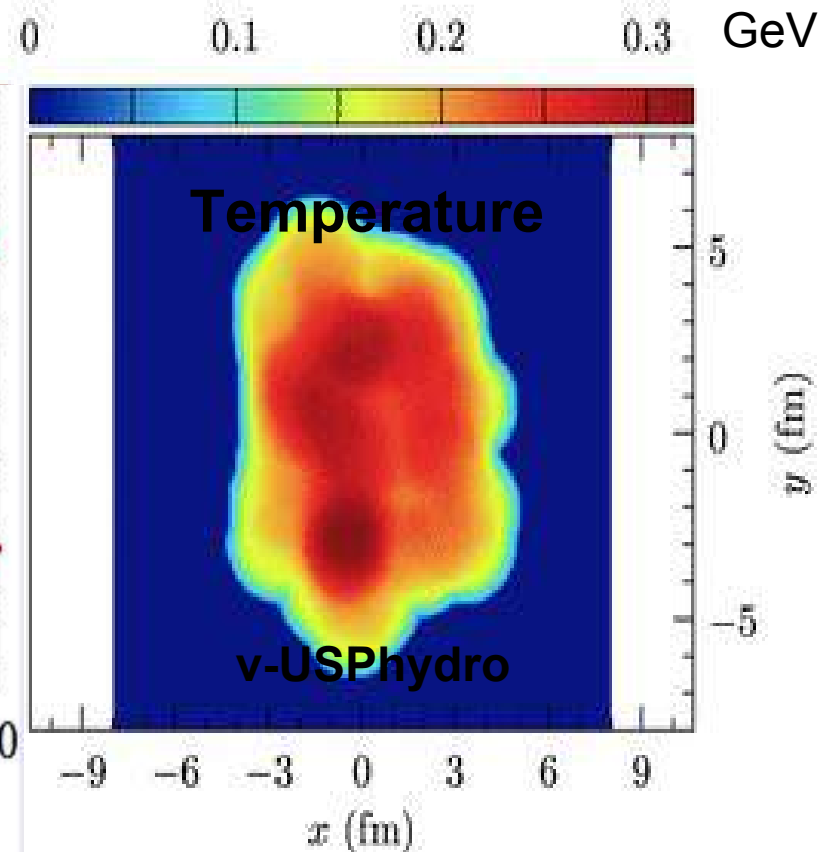
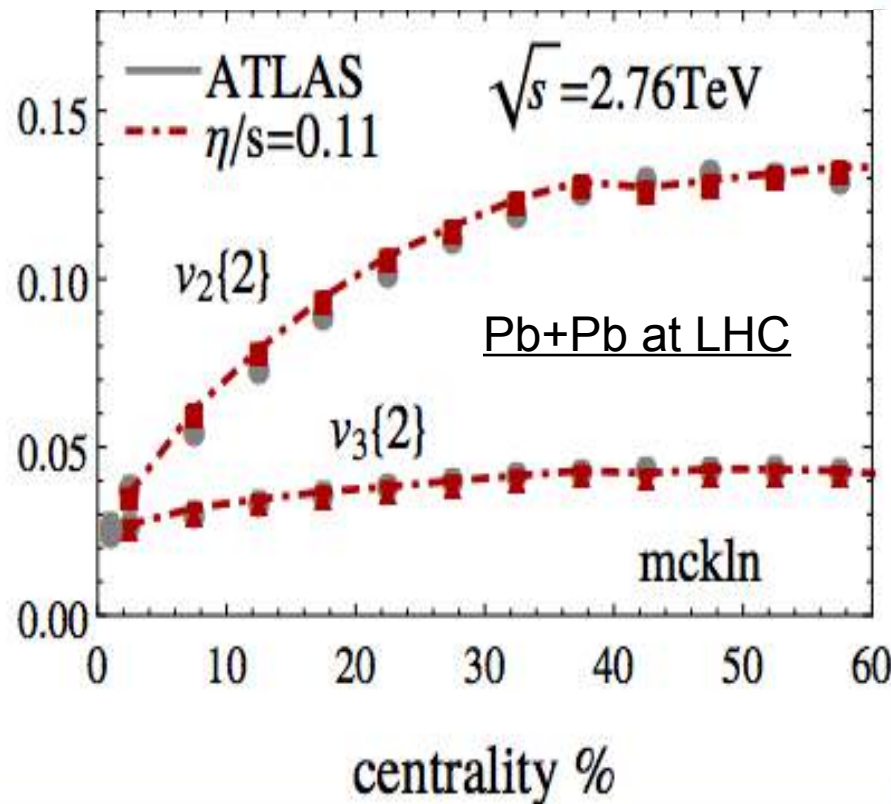
$n = 5$

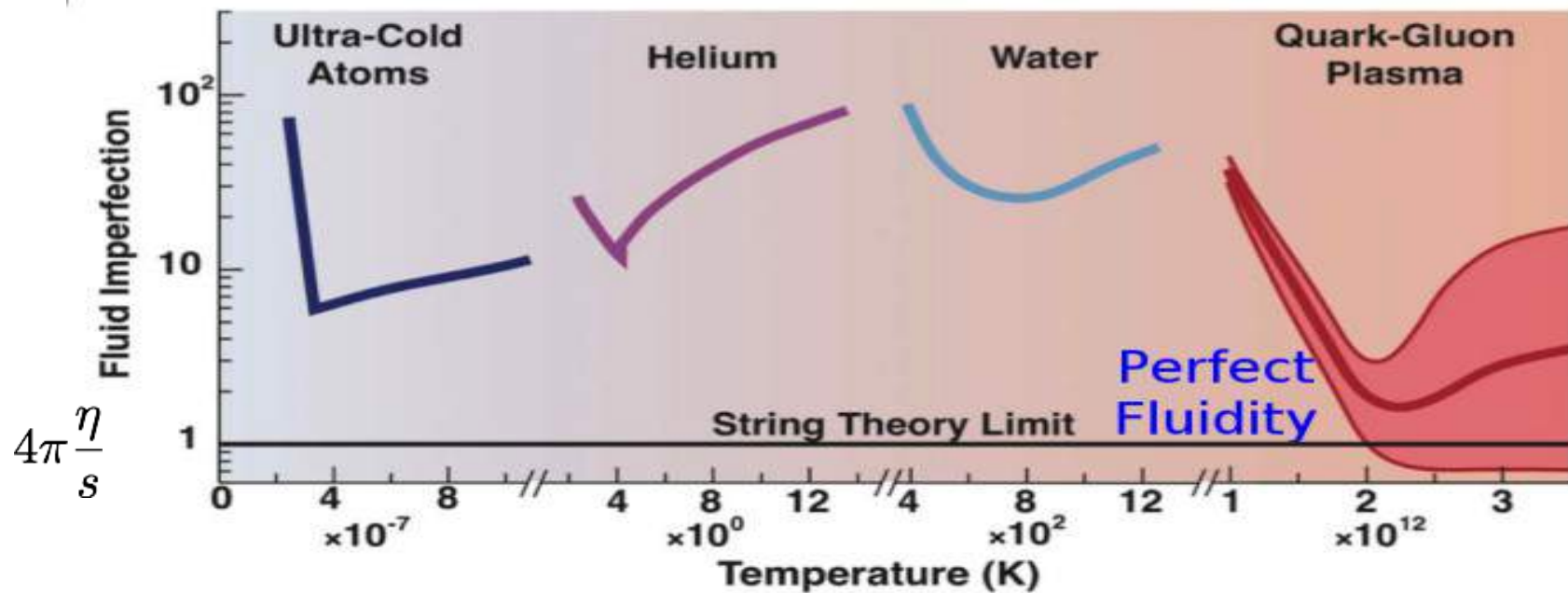
(Nearly) Perfect fluidity: an emergent property of QCD

QGP behaves as a strongly coupled relativistic fluid !!!

$$\eta/s \sim 0.05 - 0.2$$

Noronha-Hostler, Betz, JN, Gyulassy, PRL 2016

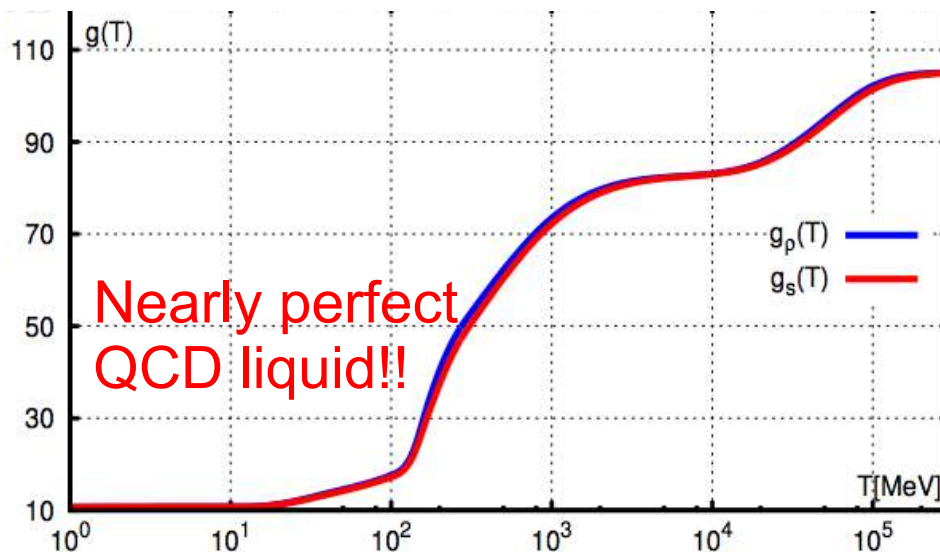




QGP = Primordial liquid microseconds after the Big Bang

Equation of state of the early universe

Borsanyi et al.
Nature 539 (2016)



Early universe
filled with perfect
liquid!

Consequences?

But how does a fluid behave near the speed of light???

Ideal fluid

$$\frac{\eta}{s} = 0$$

viscous fluid

$$\frac{\eta}{s} = 0.16$$

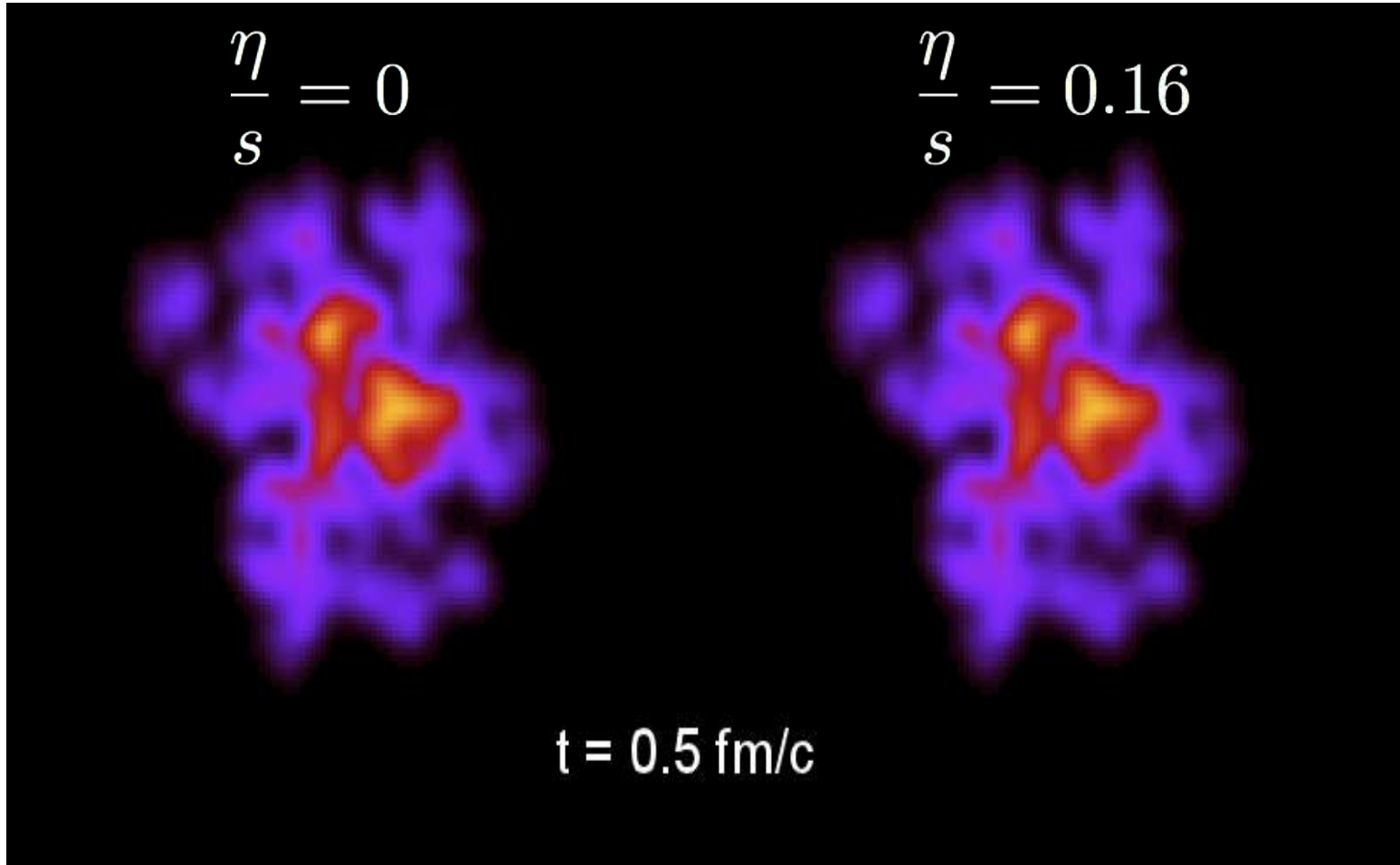


Fig. by B. Schenke

Fluid Dynamics + Special Relativity

Conservation laws
(energy and momentum)

$$\nabla_{\mu} T^{\mu\nu} = 0$$

$$T^{\mu\nu} = \begin{bmatrix} \text{energy density} & \text{energy flux} & & \\ T_{00} & T_{01} & T_{02} & T_{03} \\ \text{momentum density} & \text{momentum flux} & & \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}$$

shear stress
pressure

Energy-momentum tensor

$$T^{\mu\nu} = \underbrace{(\varepsilon + P(\varepsilon))u^{\mu}u^{\nu} + P(\varepsilon)g^{\mu\nu}}_{\text{ideal part}} + \underbrace{\pi^{\mu\nu}}_{\text{dissipative part}}$$

What is the dissipative part?

Standard view (past 100 years): Gradient expansion

Hydrodynamics → Effective theory for $\{T, u_\mu\}$ near local equilibrium

Include all possible contributions allowed by symmetries

$$T^{\mu\nu} = \underbrace{T_{ideal}^{\mu\nu}}_{\text{Zero viscosity}} + \underbrace{\mathcal{O}(\partial^\mu u^\nu)}_{\text{1st order Kn Navier-Stokes}} + \underbrace{\mathcal{O}(\partial^2 u, \partial^2 T)}_{\text{2nd order Kn}^2 \text{ (BRSSS)}} + \dots$$

Relativistic hydrodynamics as a derivative expansion

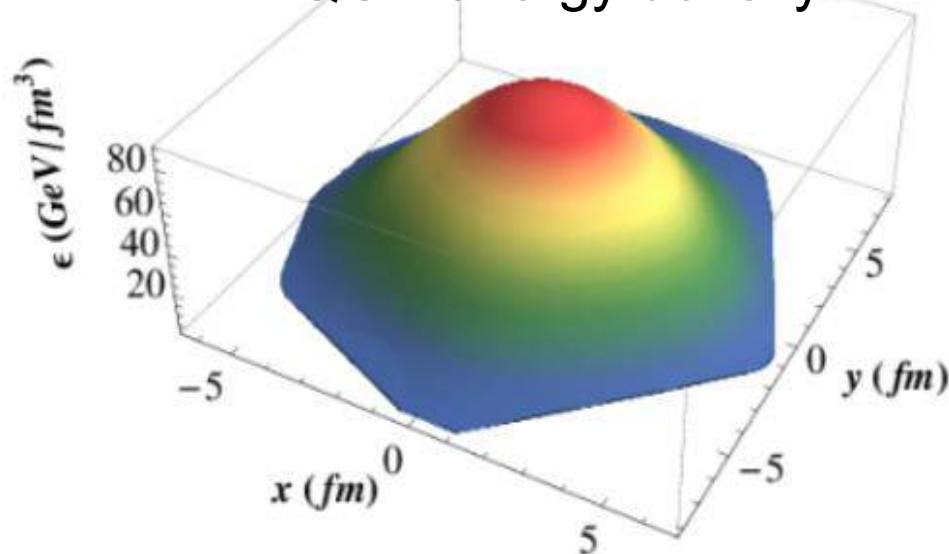
Relativistic Navier-Stokes (1940's)

$$\pi^{\mu\nu} = \underbrace{-\eta\sigma^{\mu\nu}}_{1^{\text{st}} \text{ order}} + \underbrace{\mathcal{O}(\partial^2 \varepsilon, \partial^2 u)}_{\text{assumed to be small}}$$

For simplicity
 $\zeta \rightarrow 0$

When would this be a good approximation?

QGP energy density



$L \sim$ size of large nucleus

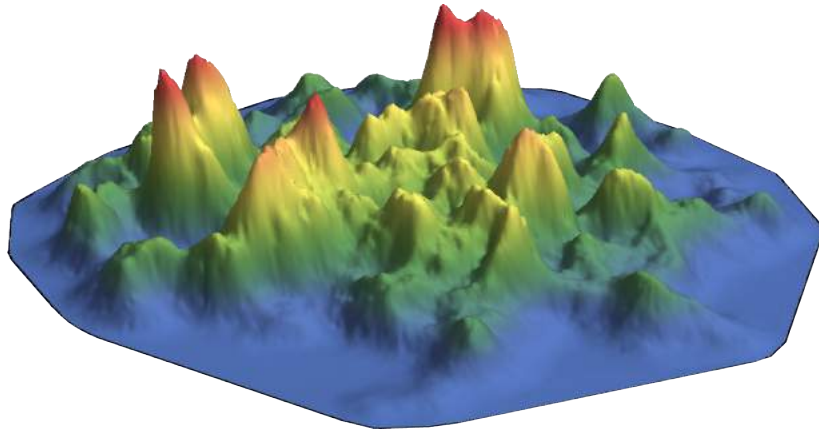
$\ell \sim 1/T \sim 1 \text{ fm}$

Knudsen number

$$K_N \sim \ell/L < 0.1$$

Reality is much more complicated ...

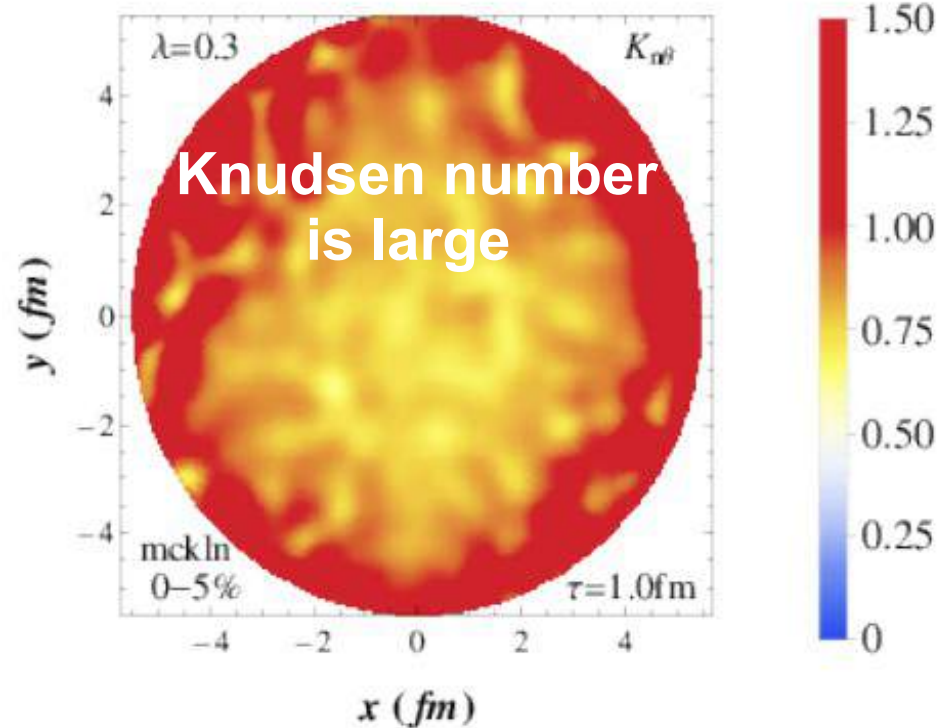
QGP energy density



J. Noronha-Hostler, B. Betz, JN, M. Gyulassy
PRL 2016

Large initial state color fluctuations

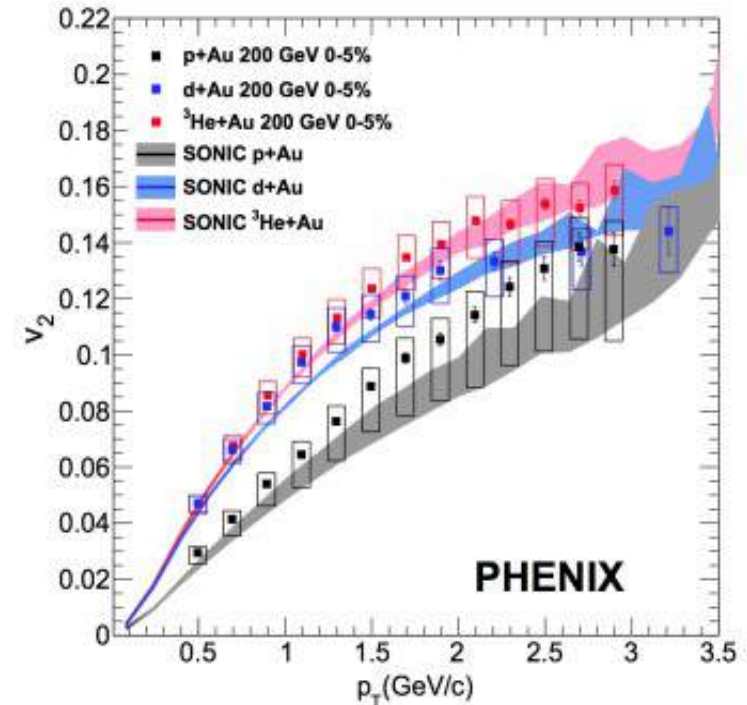
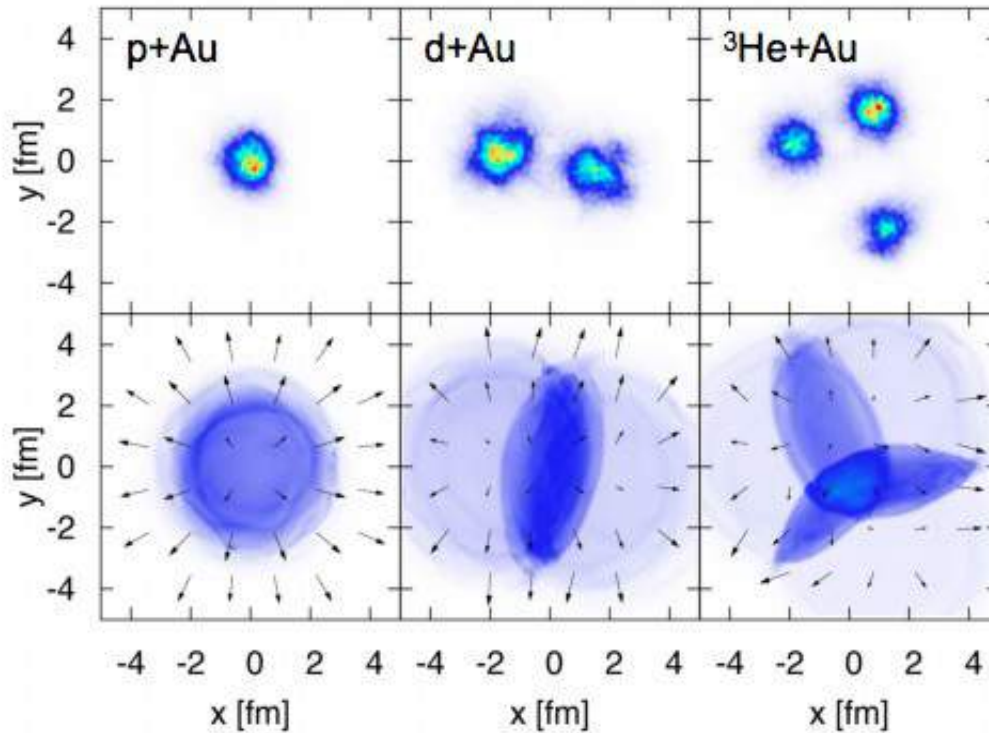
J. Noronha-Hostler, JN, M. Gyulassy, PRC 2016



PARADOX: Knudsen number not small but hydro still works

How can one push this even further?

What is the smallest droplet of QCD liquid?

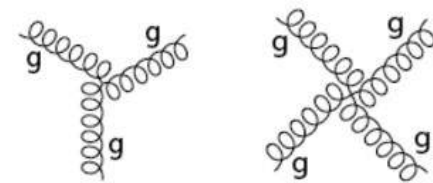
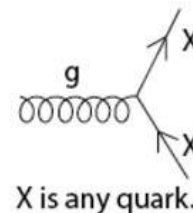


Aidala et al. PRC 2017

Nagle et al. PRL 2014, see also Phenix Collab., Nature Physics 2018

Collective behavior at scales of the size of the proton !

How does liquid behavior emerge from QCD

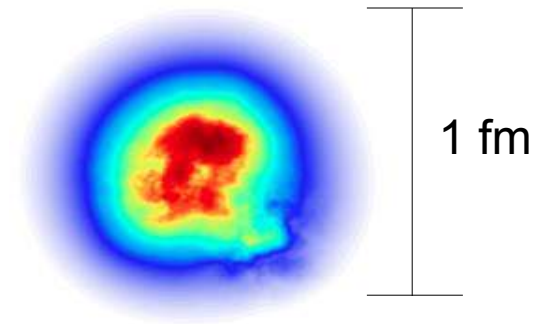


gluon self-interactions

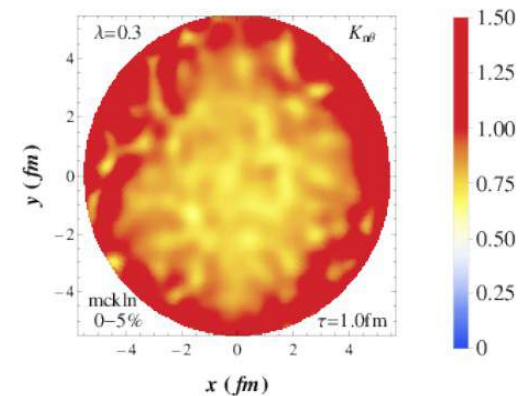
?

Challenges to the foundations of fluid dynamics

1) Liquid-like behavior of the proton?

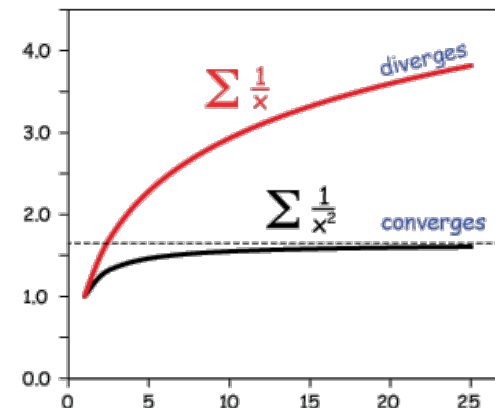


2) What happens when $K_N \sim 1$?



3) Does the Knudsen series converge?

Non-perturbative terms?



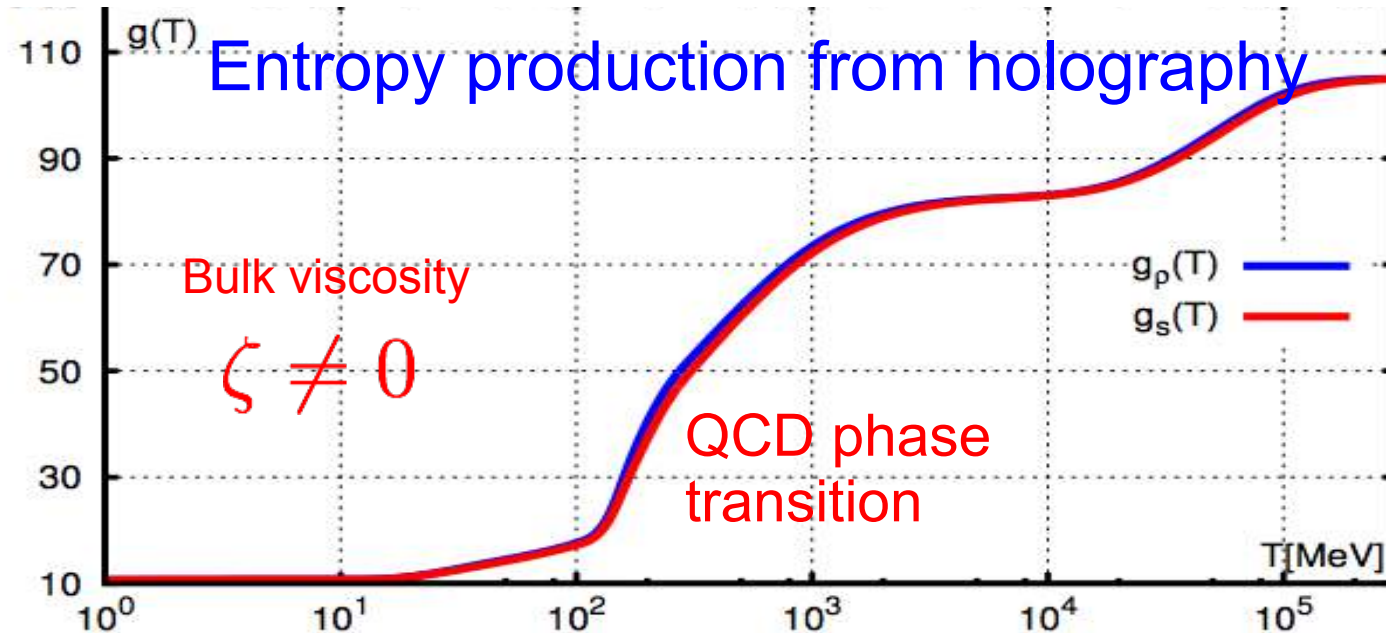
Far-From-Equilibrium Hydrodynamics

From paradox to paradigm



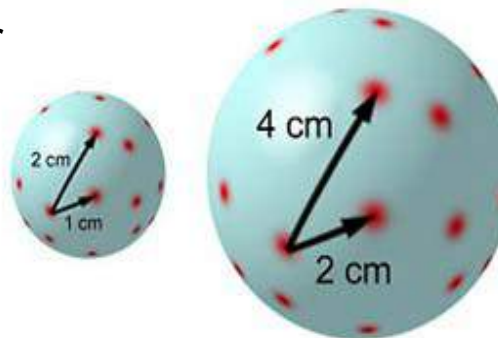
QCD phase transition in the early universe

Buchel, Heller, JN, arXiv:1603.05344, PRD (2016)



Embed non-conformal strongly coupled plasma ($N=2^*$)

Friedmann-Robertson-Walker
(FRW) universe



Knudsen number

$$K_N \sim H \sim \nabla_\mu u^\mu$$

QCD phase transition in the early universe

Buchel, Heller, [JN](#), arXiv:1603.05344, PRD (2016)

Entropy production from holography

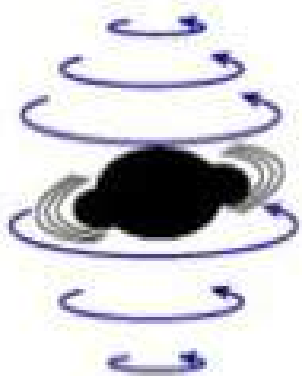
$$\frac{dS}{dt} \sim \left[\sum_{n=0}^{\infty} c_n (K_N)^n \right]^2$$

$$c_n \sim n!$$

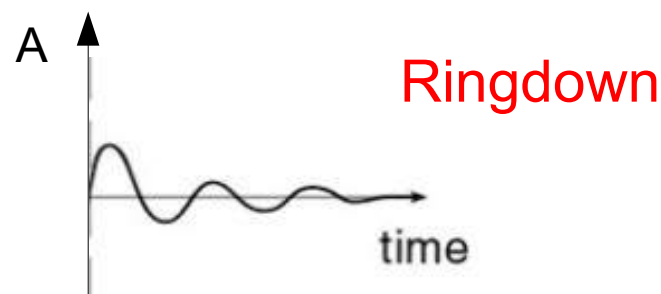
Series diverges!

After resummation of the divergent series $\rightarrow \sim e^{-\omega_{QNM}/K_N}$

Dual to black hole



Quasinormal mode



“Divergent series are the invention of the devil, and it is shameful to base on them any demonstration whatsoever”

N. H. Abel (1802-1829)

Resurgence theory

Ecalte (1980), Dunne, Unsal, Schiappa, Heller, Spalinski, Basar, Aniceto, and etc

From PARADOX to PARADIGM:

Hydrodynamics beyond the gradient expansion

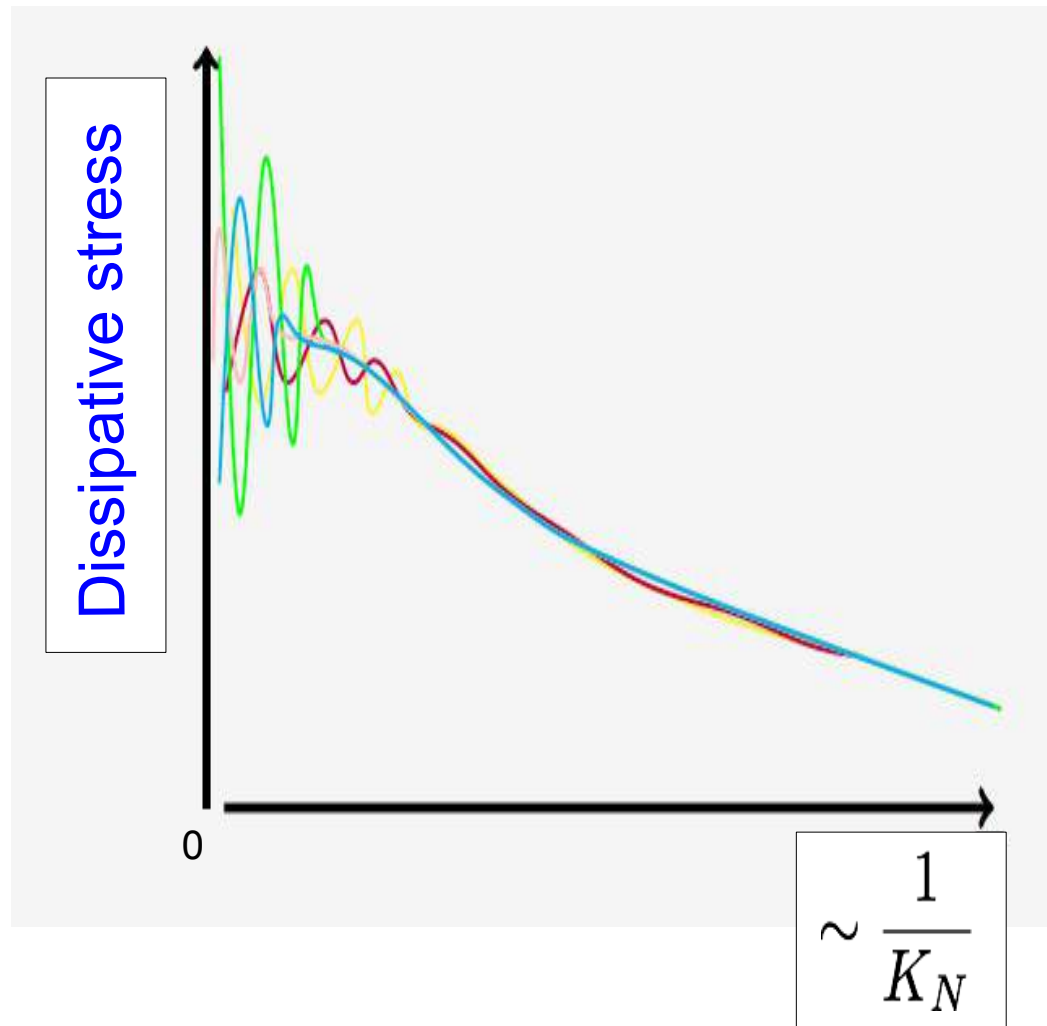
$$\pi^{\mu\nu} \sim \sum_{n=0}^{\infty} c_n^{(0),\mu\nu} (K_N)^n + c^{(1),\mu\nu} (K_N)^\beta e^{-S/K_N} + \dots$$

hydrodynamic “instanton”

Universal behavior in gauge theories far-from-equilibrium ?

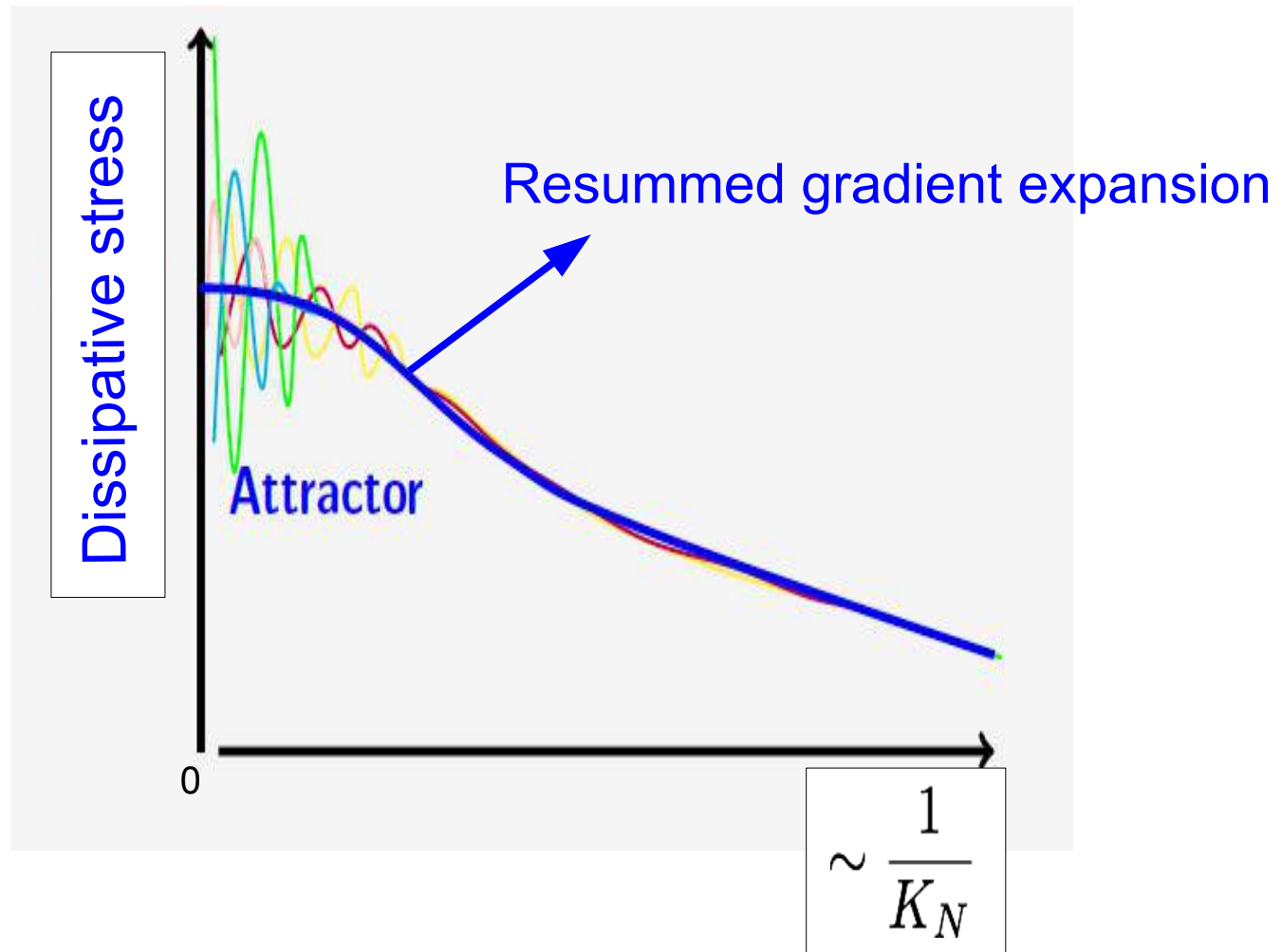
Universal behavior far-from-equilibrium

Kinetic theory and Holography

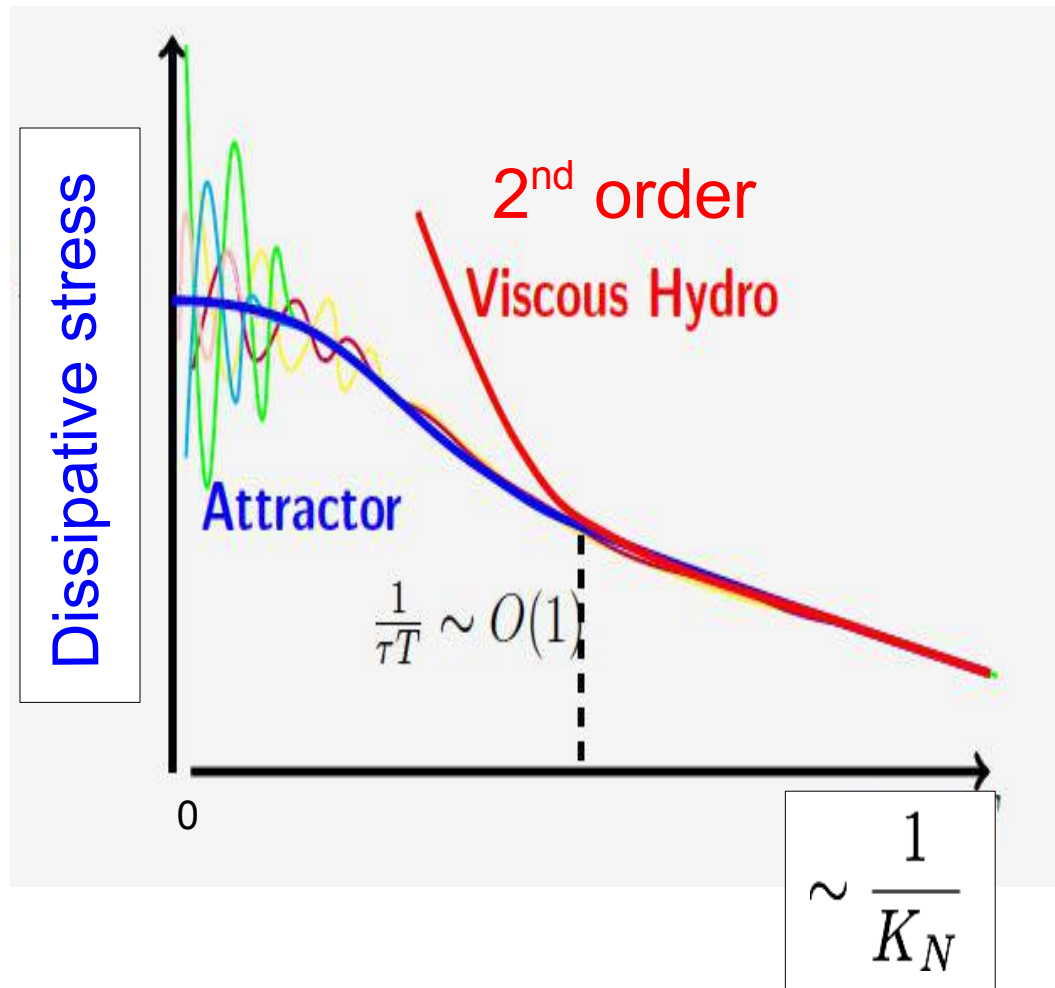


Far-from-equilibrium hydrodynamics

Emergence of constitutive relations far-from-equilibrium



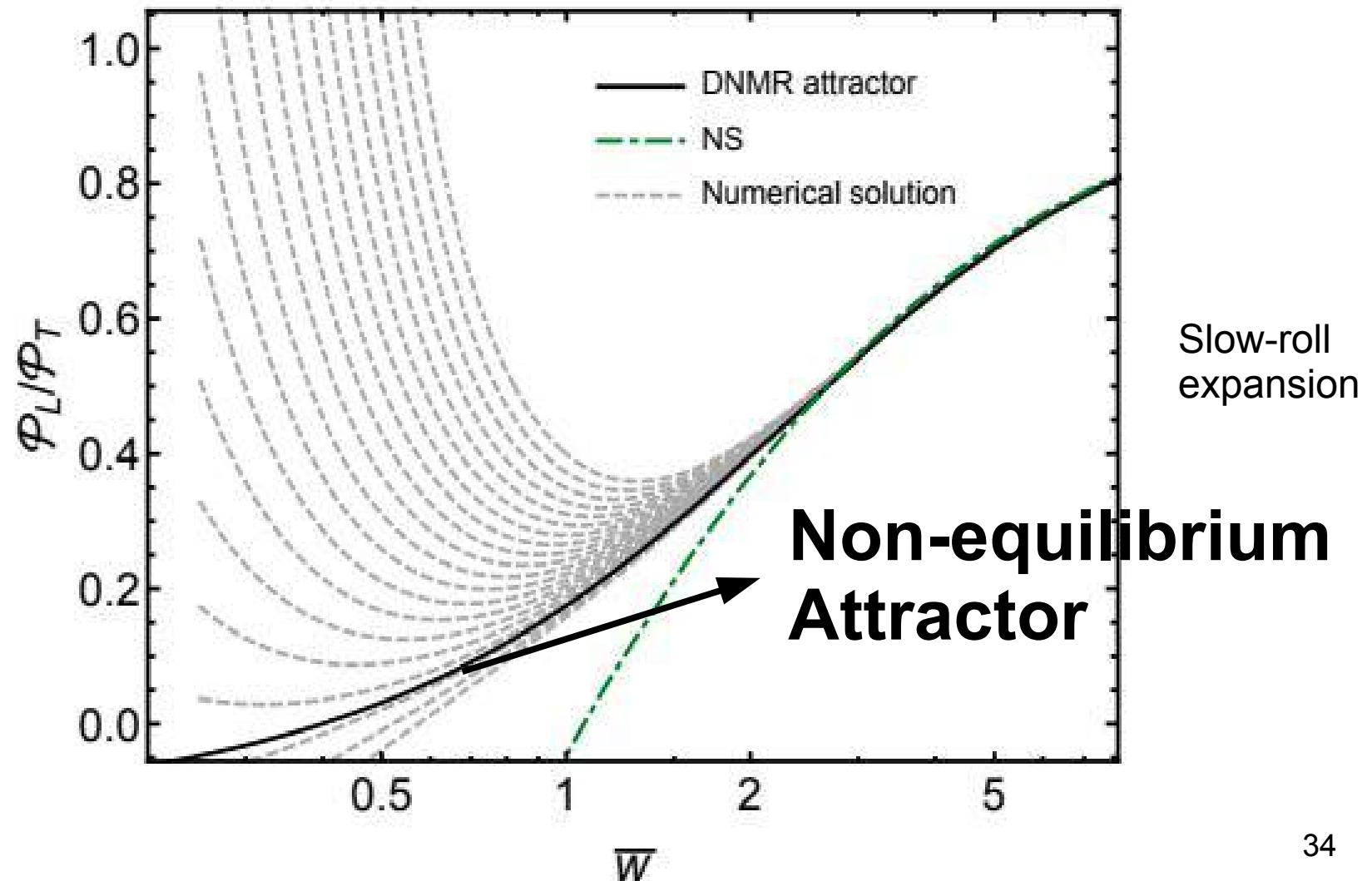
Far-from-equilibrium hydrodynamics



Details of the attractor vary between weak and strong coupling

Emergence of universal behavior far-from-equilibrium

M. Strickland, JN, G. Denicol, PRD 2018



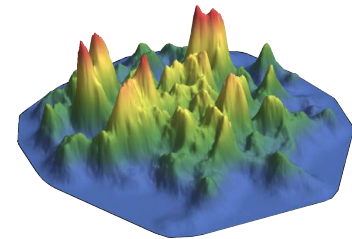
A partial list of the challenges one faces here

- Real fluids have many Knudsen numbers (fields).

Ex: Imagine trying to find leading terms in resurgent series in QFT with many couplings

- Resummation machinery (likely) not useful for realistic flows.

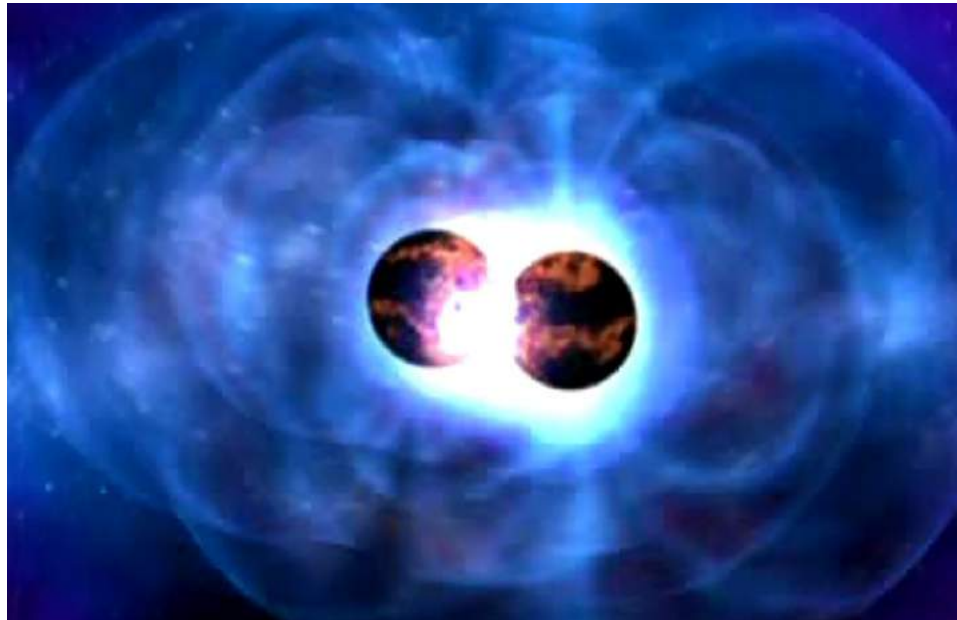
*A solution is on the horizon – ask me after the talk.



- Effective theory description for the non-equilibrium attractor?

* Fluctuation/dissipation theorem?

How does a lump of baryon rich matter behave under strong gravitational fields?



Neutron Star Mergers

High Density QCD Matter: From the Lab to the Sky

Neutron Star Mergers

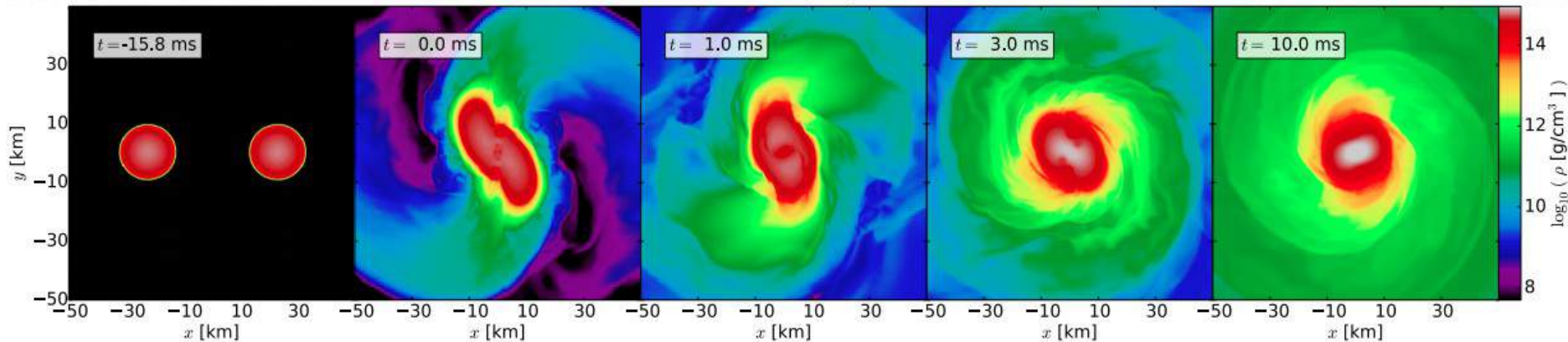


Fig. by L. Rezzolla

Viscous fluid dynamics + strong gravitational fields?

Open problem in physics and mathematics (since 1940)

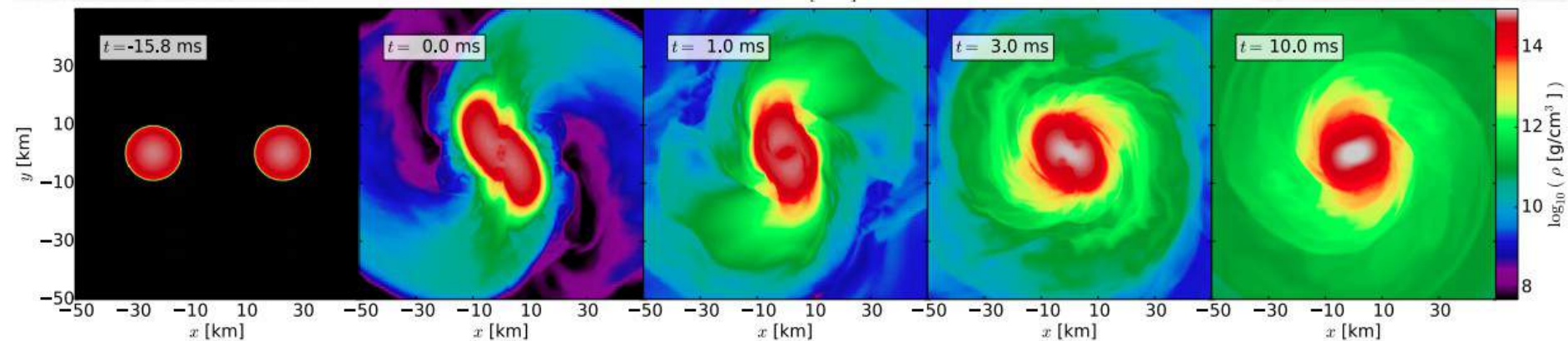
Viscous effects in neutron star mergers?

Duez et al PRD (2004), Shibata et al. PRD (2017), Alford et al. PRL (2018)

Viscous effects in binary neutron-star mergers?

Current assumption (since 1992): viscous effects do not matter

Bildsten and Cutler, Astrophys. J. (1992)



Why? Based on the simulations/knowledge at that time:

- Transport time scales estimated to be far from $\sim$ milliseconds
- Temperatures not so large, system very smooth, gradients too small

Viscous effects in binary neutron-star mergers

Exceptions: Duez et al PRD (2004), Shibata et al. PRD (2017)

Alford, Bovard, Hanauske, Rezzolla, Schwenzer, PRL (2018):

Post-merger phase

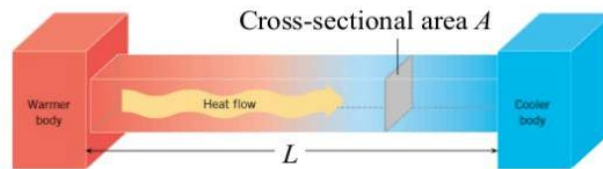
Shear dissipation:



Relevant for trapped neutrinos if $T > 10$ MeV and gradients at small scales ~ 0.01 km (e.g, turbulence).

Thermal transport:

“heat conductivity”



Relevant for trapped electron neutrinos if $T > 10$ MeV and gradients ~ 0.1 km (e.g., turbulence).

Viscous effects in binary neutron-star mergers

Alford et al. PRL (2018)

Bulk viscosity:

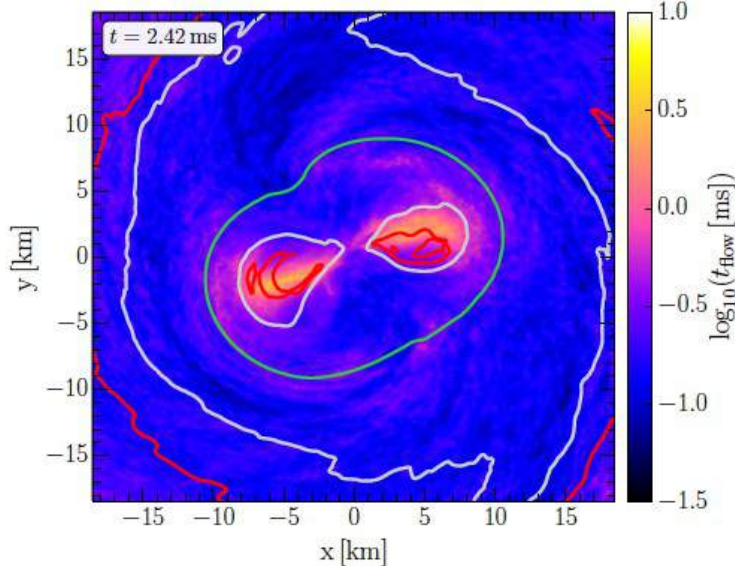
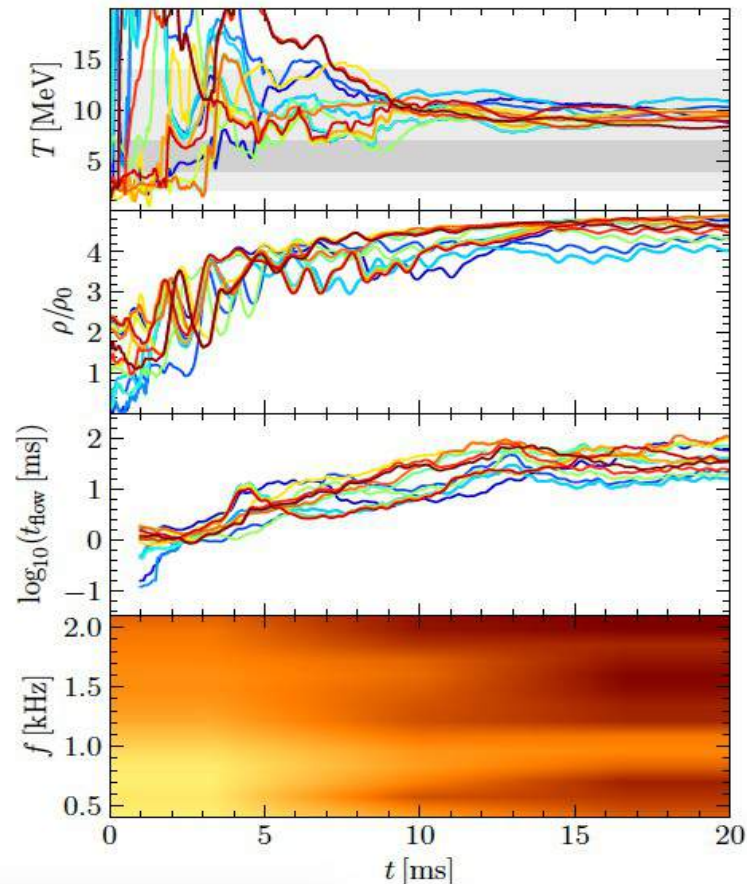


Figure 3: The flow timescale t_{flow} obtained from a numerical-relativity simulation of two $1.35 M_{\odot}$ neutron stars [40]. The red (4 MeV) and gray (7 MeV) contours show the boundaries of the temperature range in which the bulk viscosity roughly takes its maximum value, while the green contour shows the inner region where the rest-mass density exceeds nuclear saturation density.

~~If suppressed~~
 $B_1 \rightarrow B_2 + \ell + \bar{\nu}_\ell$;
 $B_2 + \ell \rightarrow B_1 + \nu_\ell$,

Should affect density oscillations after merger!!!



Viscous effects in binary neutron-star mergers

Alford et al. PRL (2018)

“The effects of bulk viscosity should be consistently included in future merger simulations. This has not been attempted before and requires a formulation of the relativistic-hydrodynamic equations that is hyperbolic and stable”.

Challenge: Prove that the solutions $\{g_{\mu\nu}, T_{\mu\nu}, J_B^\mu\}$ are **well posed** (existence, uniqueness) and **causal in the nonlinear regime**

Einstein's equations

$$R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Conservation laws

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\nabla_\mu J_B^\mu = 0$$

+ Bulk Viscosity



Eckart
1940

Relativistic Navier-Stokes equations

Energy-momentum tensor

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu + (P - \zeta \nabla_\alpha u^\alpha) \Delta^{\mu\nu}$$

Conservation laws:

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu (n_B u^\mu) = 0$$

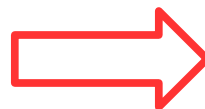


Landau
1950's

This theory is **acausal** also **unstable**
(see Pichon, 1965) (Hiscock, Lindblom, 1984)

Navier-Stokes ~ Nonlinear diffusion

$$\partial_t u = \frac{\zeta}{sT} \nabla^2 u + \dots$$



Israel-Stewart
(1970's)

- 16 coupled nonlinear PDE's (Einstein + Israel-Stewart)

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu + (P + \Pi) \Delta^{\mu\nu} \quad \Pi \longrightarrow \text{Bulk scalar}$$

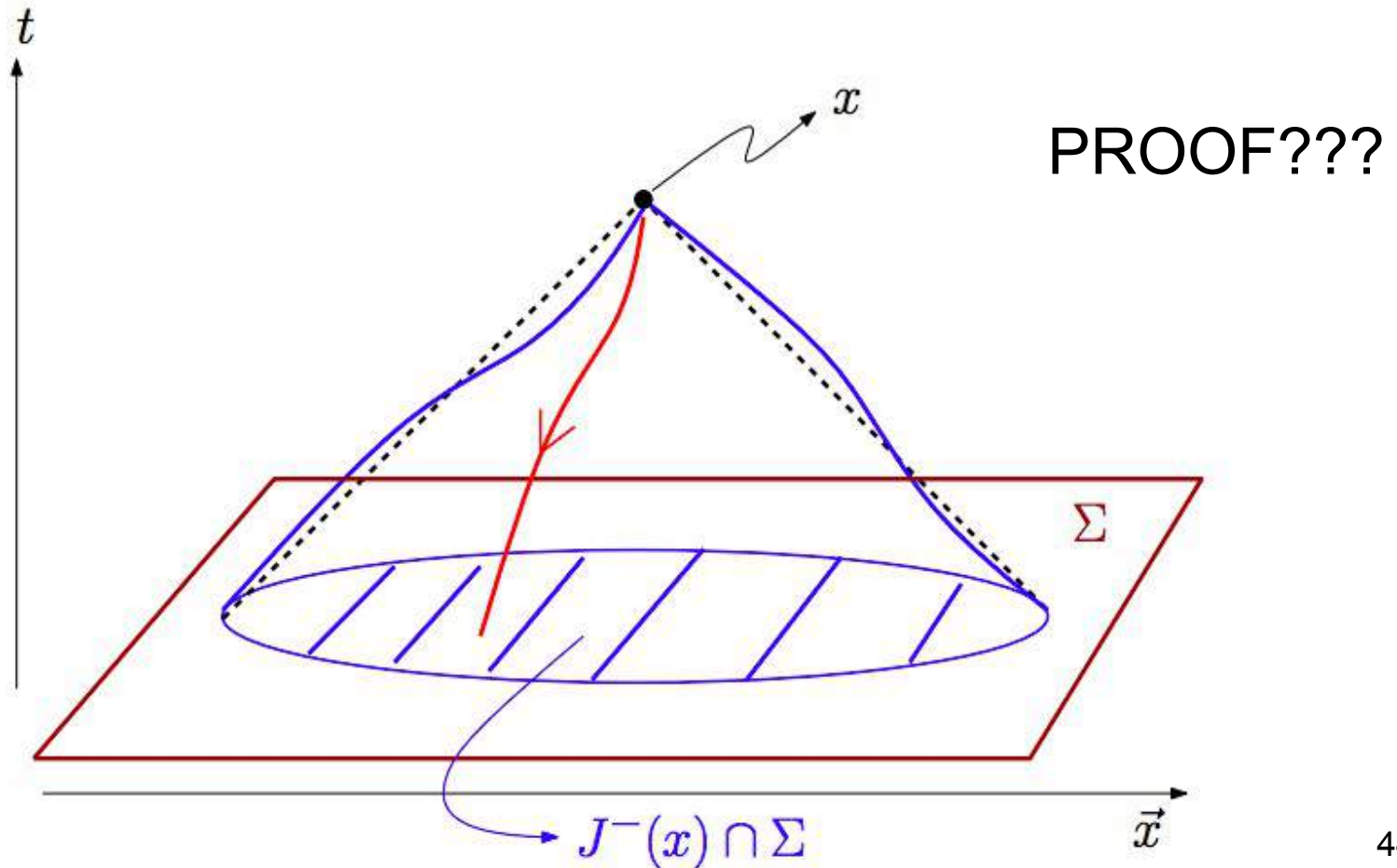
Israel-Stewart equation

$$\tau_\Pi u^\alpha \nabla_\alpha \Pi + \Pi + \lambda \Pi^2 + \zeta \nabla_\alpha u^\alpha = 0$$

- Causality in the nonlinear regime $\rightarrow$ open problem.
- Nonlinearity in hydrodynamics is notoriously hard to handle.
- Mathematical lore: Nonlinear Israel-Stewart equations seem to be beyond the reach of current mathematical techniques ...

Why is this so hard to do?

Causality in the nonlinear regime in curved spacetime?



Einstein-Israel-Stewart equations can be written as

$$A_0(\Phi)\nabla_t\Phi + A^i(\Phi)\nabla_i\Phi + B(\Phi)\Phi = 0$$

where $\Phi = (\varepsilon, n, u_\mu, \Pi, g_{\mu\nu}, \partial g_{\mu\nu})$

1st Proof of CAUSALITY, EXISTENCE, AND UNIQUENESS in the full nonlinear regime

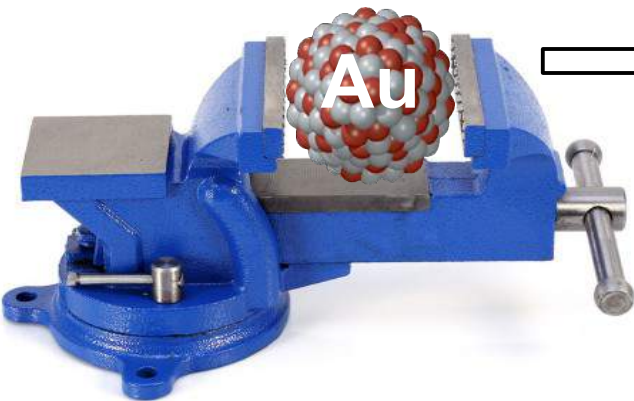
Theorem 1. *Let $(\varepsilon, u^\alpha, \Pi, n, g_{\alpha\beta})$ be a solution to the EIS equations defined on a globally hyperbolic spacetime M , and let Σ be a Cauchy surface. Suppose $\frac{\zeta}{\tau_\Pi(\varepsilon+P+\Pi)} + \alpha_1 + \frac{\alpha_2 n}{\varepsilon+P+\Pi} \geq 0$ and that (1) holds. Then, for any $p \in M$ in the future of Σ , $(\varepsilon(p), u^\alpha(p), \Pi(p), n(p), g_{\alpha\beta}(p))$ depends only on $(\varepsilon, u^\alpha, \Pi, n, g_{\alpha\beta}, \kappa_{\alpha\beta})|_{\Sigma \cap J^-(p)}$, where $J^-(p)$ is the causal past of p and κ is the extrinsic curvature of Σ in M .*

Theorem 2. *Let $\mathcal{I} = (\Sigma, \dot{\varepsilon}, \dot{u}^i, \dot{\Pi}, \dot{n}, \dot{g}_{ij}, \dot{\kappa}_{ij})$ be an initial data set for the EIS system, with an equation of state $P = P(\varepsilon, n)$, a bulk viscosity $\zeta = \zeta(\varepsilon, n)$, and a relaxation time $\tau_\Pi = \tau_\Pi(\varepsilon, n)$. Assume that $\dot{\varepsilon} + P(\dot{\varepsilon}, \dot{n}) + \dot{\Pi}$, $\tau_\Pi(\dot{\varepsilon}, \dot{n})$, $\zeta(\dot{\varepsilon}, \dot{n}) > 0$, $\frac{\partial P}{\partial \varepsilon}(\dot{\varepsilon}, \dot{n}) + \frac{\partial P}{\partial n}(\dot{\varepsilon}, \dot{n})\dot{n}/(\dot{\varepsilon} + P(\dot{\varepsilon}, \dot{n}) + \dot{\Pi}) \geq 0$, and that $\dot{n}$, $\frac{\partial P}{\partial \varepsilon}(\dot{\varepsilon}, \dot{n})$, and $\frac{\partial P}{\partial n}(\dot{\varepsilon}, \dot{n})$ are nonzero. Suppose that $\dot{g}_{ij} \in H_{ul}^{s+1}(\Sigma)$, $\dot{\varepsilon}, \dot{u}^i, \dot{\Pi}, \dot{n}, \dot{\kappa}_{ij} \in H_{ul}^s(\Sigma)$, and that $P, \zeta, \lambda, \tau_\Pi \in C^s(\mathbb{R}^2)$, where $s \geq 3$. Suppose that (1) holds for $\mathcal{I}$. Then, there exists a globally hyperbolic development of $\mathcal{I}$.*



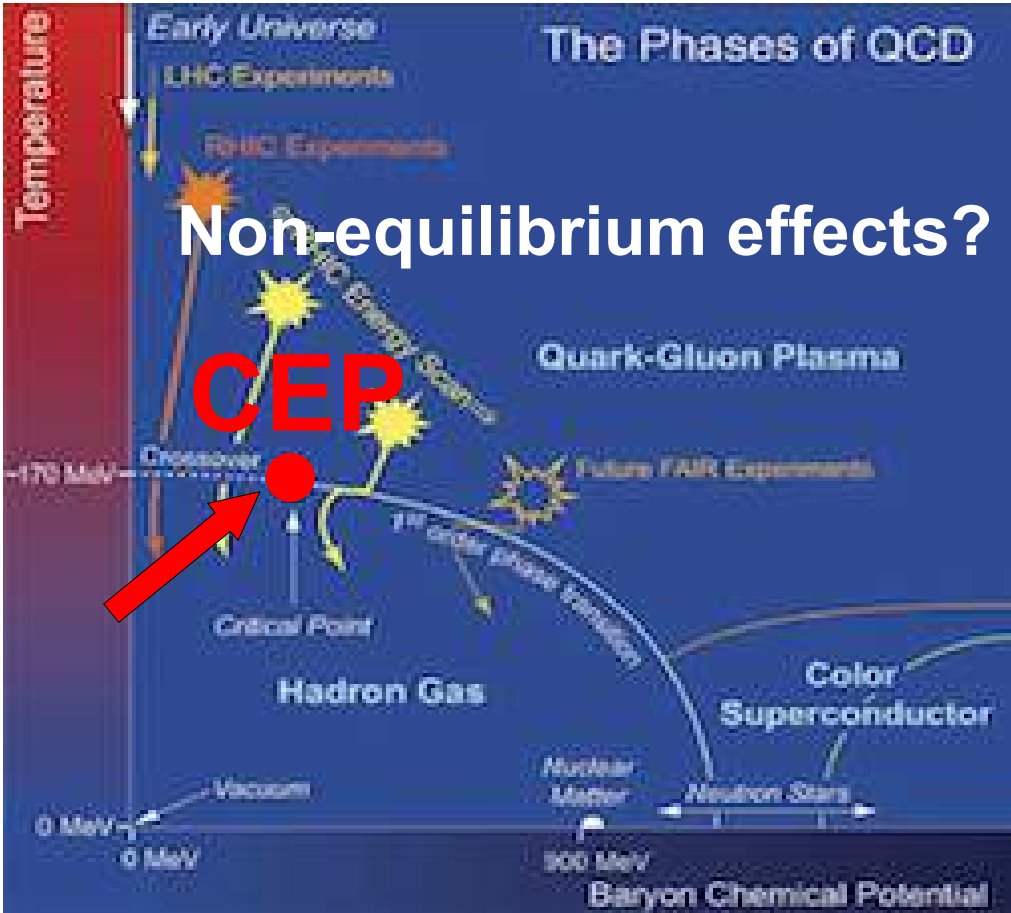
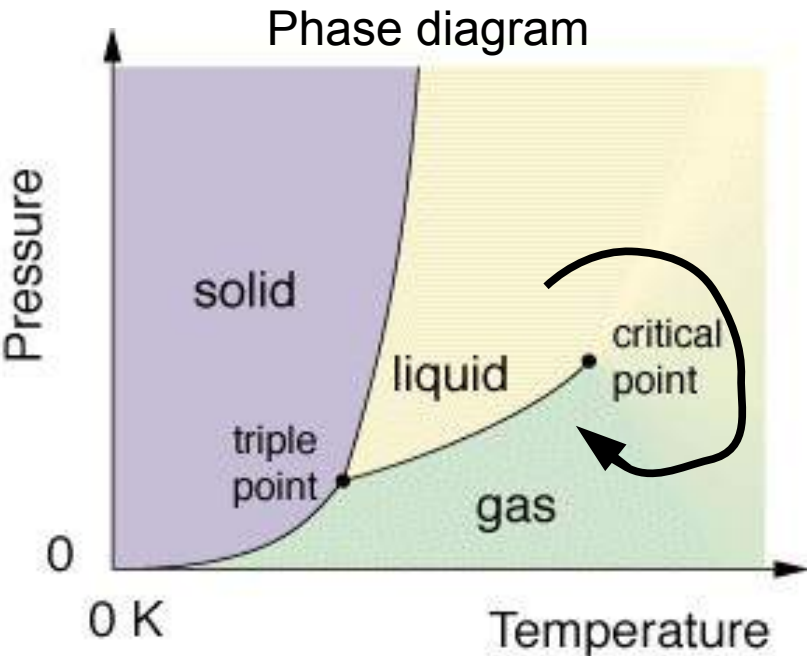
Frontiers of High Energy QCD through the next decade

2019+ → QCD at large baryon densities (**RHIC**, FAIR, NICA)



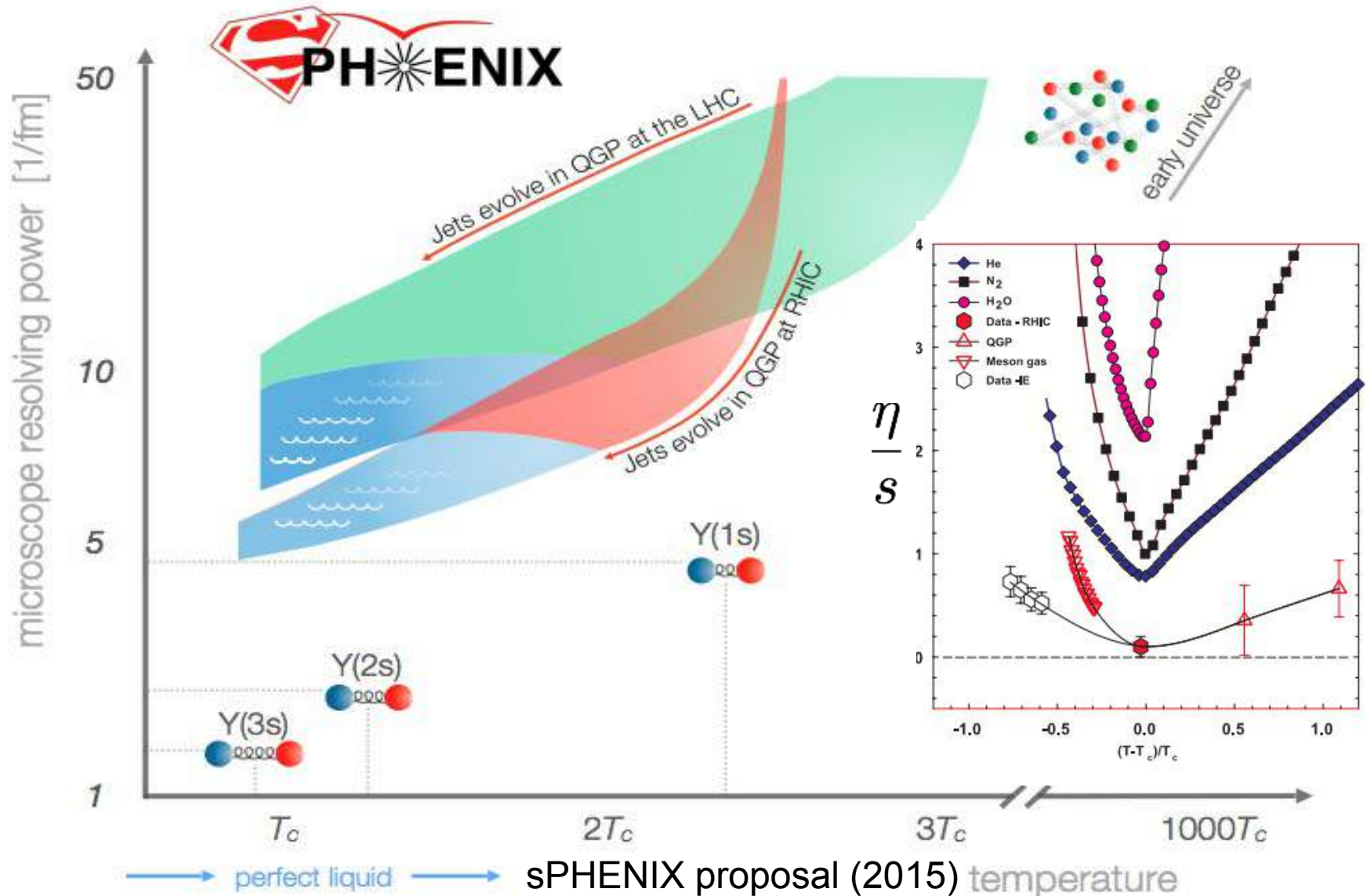
→ Signatures of critical phenomena in QCD?

RHIC Beam Energy Scan (BES) II

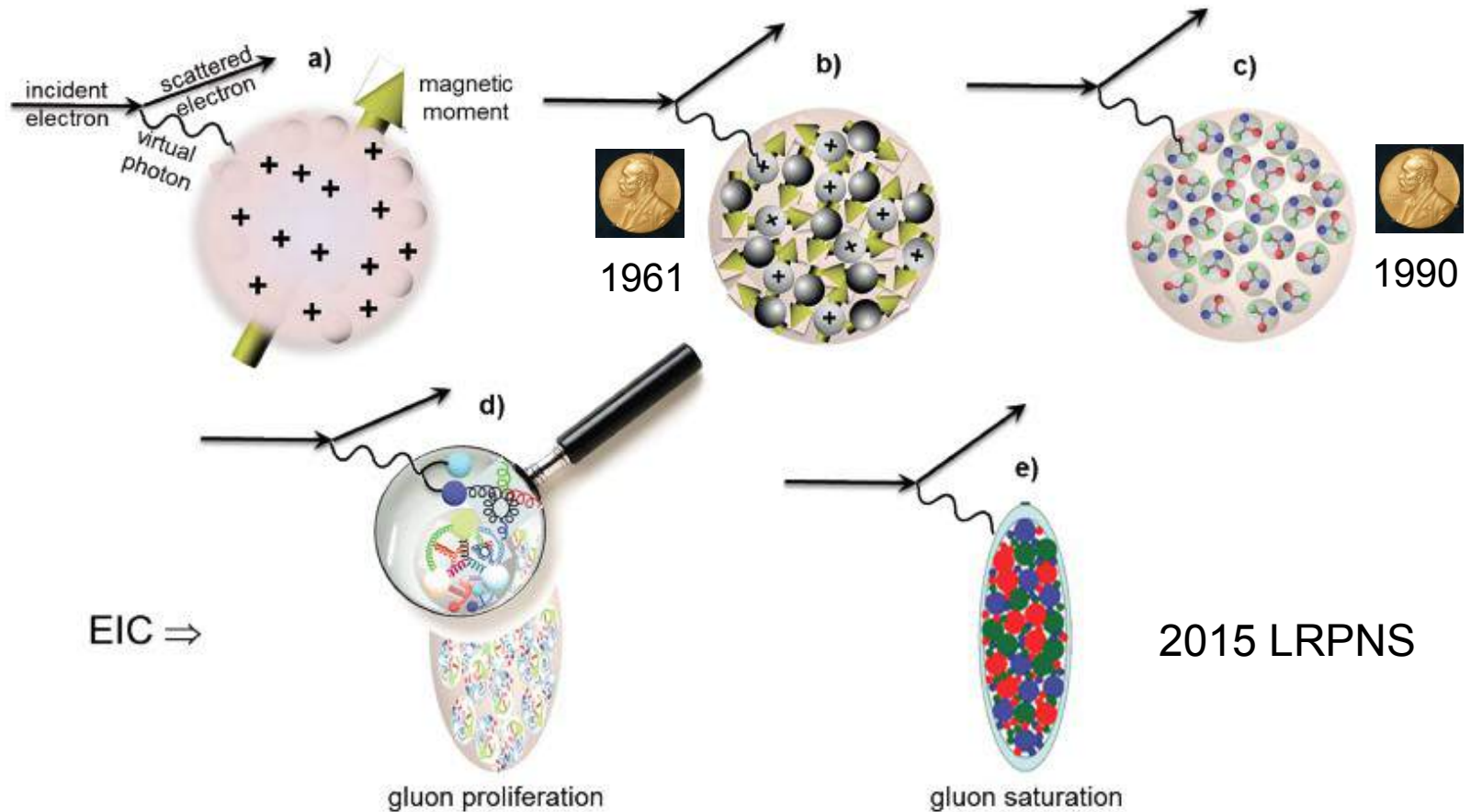


2020+ → Jetting through the QGP (LHC, sPHENIX) Jets in AA, pA

RHIC



2030+ → Electron Ion Collider



Nucleon
Spin

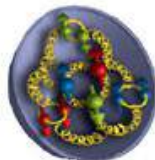
$$\frac{1}{2} =$$

Spin of all
Quarks



+

Spin of
Gluons



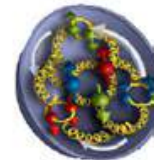
+

Angular Momentum
of all Quarks



+

Angular Momentum
of Gluons



Conclusions

- QGP was nature's first and most perfect liquid.
- Liquid-like behavior → emergent property of QCD.
- Many challenges to the foundations of fluid dynamics.
- Many connections to string theory, cosmology, astrophysics, and mathematics.
- Novel viscous effects in neutron star mergers.
- Next frontier in high energy QCD (LHC, BES-RHIC, FAIR, sPHENIX, EIC).